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ABSTRACT

Over-the-horizon radar (OTHR) and microwave radar networks can together generate track data over a wide surveillance region. However the data is often subject to ambiguity and uncertainty due to the complexities of the HF signal propagation environment, which give rise to multipath OTHR tracks, as well as ambiguities in target associations between multiple microwave radars. This report describes an association and fusion algorithm which deals with both sources of uncertainty. The algorithm is capable of fusing OTHR multipath tracks and non-OTHR tracks (e.g. microwave radar or GPS), as well as dealing with multipath tracks from OTHR networks. The algorithm achieves this through a very general, model based, approach which deals with multipath effects as well as asynchronicity between sources of data. Importantly, the approach incorporates track history in its computation of association probabilities and fused estimate calculations, thus exploiting temporal as well as instantaneous spatial relationships between tracks.

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EXECUTIVE SUMMARY

The use of over-the-horizon radar (OTHR) in conjunction with microwave radar (MR) networks enables surveillance over a large region of land/sea, with potentially greater reliability and accuracy in target acquisition and tracking over a single radar. For example, the accuracy and clarity of OTHR data can be enhanced by fusing OTHR tracks with associated microwave radar tracks which are known with greater precision and are not subject to multipath effects of the sort that plague OTHR. To achieve this potential, substantial challenges which occur due to ambiguities and uncertainties in target associations between multiple microwave radars as well as multipath OTHR tracks, must be overcome. Multiple target tracks arise in OTHR surveillance due to multiple ionospheric propagation paths between targets and radar locations. The use of multiple microwave radars also leads to multiple tracks for single targets. Hence target track association and fusion algorithms for OTHR/MR networks are required to give a unified picture of the surveillance region with one track per target in ground coordinates.

This report describes an algorithm for the association and fusion of multipath OTHR tracks as well as microwave radar tracks which fall within the OTHR surveillance region. Both multiple microwave radars and multiple OTHRs can be accommodated by the algorithm. Global Positioning System (GPS) reports from commercial aircraft, air lanes and airfield locations could also be incorporated if required. This algorithm can be used to enhance the value of the OTHR surveillance data from the Jindalee Facility Alice Springs (JFAS) and the Jindalee Over-the-horizon Radar Network (JORN) for air picture compilation and to aid the integration of OTHR into a multi-sensor air surveillance system. At the time of writing this report, testing of the algorithm with multipath OTHR tracks has commenced on a test-bed as well as on a prototype version in an operational OTHR. The results of testing to date indicate that, at the present state of development, the algorithm can be used in a semi-automated fashion to give advice to an operator. However, higher levels of automation can be expected with further development.

Authors

Peter W. Sarunic
Surveillance Systems Division

After completing his B. Eng. degree, Peter worked as an Electrical/Electronic Engineer in private industry for 5 years. He then joined DSTO in 1986 to work in what was at the time called Radar Division (its most recent descendent at the time of writing this report being Surveillance Systems Division). His major fields of work were tracking, signal processing and radar systems engineering. While employed at DSTO, Peter completed a B.Sc. (mathematical and computer sciences) and a M.Eng. degree (electronic engineering). For his masters degree, he developed a multiple model adaptive tracking algorithm for use with electronically steerable phased array radars.

In 1996, after 10 years with DSTO, Peter moved overseas to Canada. There, he spent 2 years working on radar and data fusion problems. In 1998, he returned to DSTO where he joined TSF Group, SSD, to do research on multipath track fusion and multisensor fusion, ie, the topic of this report. Presently, Peter is working in Surveillance Sensor Processing (SSP) Group, SSD. In his current postition, he is conducting research on electronic protection for radar.

K.A.B. White
Surveillance Systems Division

Kruger White obtained a Bachelor of Applied Science in Applied Physics from the South Australian Institute of Technology in 1988. In 1989 he majored in Physics and obtained a Bachelor of Science (First class Honours) from the Flinders University of South Australia. A PhD was then undertaken at Flinders University. For his PhD, Kruger investigated the physical process of driving a steady-state current using radiofrequency electromagnetic fields travelling in a helical manner around a torus that contained an ionised gas known as plasma.

In 1994 Kruger joined the then High Frequency Radar Division of DSTO to work in the area of data fusion. In particular he conducted research and developed prototype code for performing the fusion of multipath tracks arising from Over-The-Horizon Radar. Other activities include participation in joint US-Australia Radar Agreement meetings between organisations with an interest in Over-The-Horizon Radar technologies, and liaison with research organisations such as the Cooperative Centre for Sensor Signal and Information Processing and Melbourne University.

In January 2000, Kruger White was appointed as the Multisensor Integration (MSI) Scientist with the Airborne Early Warning and Control (AEW&C) Resident Project Team located with Boeing in Seattle, USA. He has responsibility for the MSI functions of tracking, track identification, situation and threat assessment and sensor management. Kruger remains attached to the Tracking and Sensor Fusion group within Surveillance Systems Division of DSTO.

M.G. Rutten
Surveillance Systems Division

Mark Rutten received the B.Sc. degree in Mathematical Science and the B.E. degree with first class honours in Electrical and Electronic Engineering in 1994 and 1995 respectively, both from The University of Adelaide. He received the M.Sc. degree in Mathematical Signal and Information Processing, also from The University of Adelaide, in 1998. Mark joined DSTO in 1996, initally working on development of an artificially intelligent agent architecture. Since 1999 he has been involved in analysis and development of multipath track-to-track fusion for Over-the-Horizon Radar. His main research interests are distributed sensor fusion and estimation theory.

Contents

Appendices

Figures

Glossary

Abbreviations:

ADF Australian Defence Force

CI Covariance Intersection

CR Coordinate Registration

DMPTF Dynamic Multipath Track Fusion

GPS Global Positioning System

JFAS Jindalee Facility Alice Springs

JORN Jindalee Over-the-horizon Radar Network

MHT Multiple Hypothesis Tracking

MPTF Multipath Track Fusion

NN Nearest Neighbour

OTHR Over-the-Horizon Radar

PDA Probabilistic Data Association

pdf Probability Density Function

SSD Surveillance Systems Division

TSF Tracking and Sensor Fusion

Mathematical Notation:

a target azimuth in ground coordinates

$A_j(k)$ azimuth component of $\psi_j(k)$

$B_{n_1..n_K}$ the number of path independent hypotheses that originate from hypothesis $\lambda_{n_1..n_K}$ in the path independent hypothesis tree; note that $B_{n_1..n_K} = N_{n_1..n_K} + 1$

$\beta_j^{m_j}$ $P\left\{\theta_j^{m_j}\right\}$; ie, $\beta_j^{m_j}$ is the *a priori* probability of $\theta_j^{m_j}$

$\delta_j(k)$ the measurements (in radar coordinates) that are used by a tracking algorithm to produce the estimates $\psi_j(k)$

$\Delta^j(k)$ the sequence of measurements $\delta_1(k), .., \delta_j(k)$

D^k all the (radar coordinate) track data available up to and including update k

$E\{.\}$ mathematical expectation

$F(k-1)$ the state transition matrix of a target for the time interval between updates $(k-1)$ and k

H measurement matrix used for transforming estimates in state space to predicted measurements (both in ground coordinates)

$J(k)$ the number of multipath tracks to be associated and fused at update k

$\lambda_{n_1 n_2 .. n_K}$ the (composite, path independent) hypothesis that τ_j are associated with targets $n_j,\ j = 1, 2, .., K,\ K \le J$. Assuming there are no false tracks, the values that can be taken by n_j are: $n_j \in \{1, 2, .., B_{n_1 .. n_{j-1}}\}$. $B_{n_1 .. n_{j-1}}$ is defined below.

$\lambda_{n_1 n_2 .. n_K}^{m_1 m_2 .. m_K}$ the (composite, path dependent) hypothesis that τ_j are associated with targets n_j, and propagation paths $m_j,\ j = 1, 2, .., K,\ K \le J$, respectively. Assuming there are no false tracks, the values that can be taken by n_j and m_j are: $n_j \in \{1, 2, .., B_{n_1 .. n_{j-1}}\}$ and $m_j \in \{1, 2, .., M_j\}$. $B_{n_1 .. n_{j-1}}$ is defined above.

${}^j\lambda_{n_j}^{m_j}$ the (single, path dependent) hypothesis that τ_j is associated with target $n_j \in \{1, 2, .., B_{n_1 .. n_{j-1}}\}$ and propagation path $m_j \in \{1, 2, .., M_j\}$

Λ likelihood function

m_j index representing the propagation path associated with track j

M_j the number of propagation paths associated with track j

n_j index representing the target associated with track j

$N_{n_1 .. n_K}$ the number of targets associated with hypothesis $\lambda_{n_1 .. n_K}$

$\mathcal{N}(x; \bar{x}, P)$ pdf of a normal (Gaussian) random vector x with mean $\bar{x}$ and covariance matrix P

$p(.)$ probability density function of a (continuous) random variable

$P\{.\}$ probability of an event

$\bar{P}_i^h(k)$ covariance matrix of $\bar{x}_i^h(k)$, ie, $\bar{P}_i^h(k) \triangleq E\{\tilde{x}_i^h(k)\,\tilde{x}_i^h(k)'\}$

$P_j^{m_j}(k)$ covariance matrix of $y_j^{m_j}(k)$, ie, $P_j^{m_j}(k) \triangleq E\left\{\tilde{y}_j^{m_j}(k)\,\tilde{y}_j^{m_j}(k)'\right\}$

$P_{ij}^{hm_j}(k)$ cross covariance matrix of $\bar{x}_i^h(k)$ and $y_j^{m_j}(k)$, ie, $P_{ij}^{hm_j}(k) \triangleq E\left\{\tilde{x}_i^h(k)\,\tilde{y}_j^{m_j}(k)'\right\}$

$P_{ji}^{m_j h}(k)$ cross covariance matrix of $y_j^{m_j}(k)$ and $\bar{x}_i^h(k)$, ie, $P_{ji}^{m_j h}(k) \triangleq E\left\{\tilde{y}_j^{m_j}(k)\,\tilde{x}_i^h(k)'\right\}$

$\phi_j^{n_j}$ the event that "track $\tau_j,\ j = 1, .., J$ is associated with target $n_j,\ n_j \in \{1, 2, .., J\}$"

$\psi_j(k)$ estimate k, in radar coordinates, of track j, where $k = 0, 1, 2, ...$

$\Psi^j(k)$ the estimates $\psi_1(k), .., \psi_j(k)$

Q_k process noise covariance matrix, ie, $Q_k \triangleq E[v(k)v(k)']$

r target range in ground coordinates

$r_j^{m_j}(k)$ range component of $y_j^{m_j}(k)$

$R_j(k)$ range component of $\psi_j(k)$

$R_j^{m_j}(k)$ covariance matrix of $z_j^{m_j}(k)$, ie, $R_j^{m_j}(k) \triangleq E\left\{\tilde{z}_j^{m_j}(k)\left(\tilde{z}_j^{m_j}(k)\right)'\right\}$

S_j the set $\{m_j : m_j = m_{j-1}$ if $n_j = n_{j-1}$, $m_j = m_{j-2}$ if $n_j = n_{j-2}, ...,$ $m_j = m_1$ if $n_j = n_1\}$

$S_j^{m_j}(k)$ covariance matrix of $\left(z_j^{m_j}(k) - \bar{z}_i^h(k)\right)$

t_i target i, $i = 1, .., T$

T the number of targets associated with a hypothesis

$T_j^{m_j}(k)$ covariance matrix of $\left(y_j^{m_j}(k) - \bar{x}_i^h(k)\right)$

τ_j track j, $j = 1, .., J$ in radar coordinates

$\theta_j^{m_j}$ the event "track τ_j, $j = 1, .., J$ is associated with propagation via propagation path m_j, $m_j \in \{1, 2, .., M_j\}$"

$U_{ij}^{hm_j}(k)$ cross covariance matrix of $\bar{x}_i^h(k)$ and $z_j^{m_j}(k)$, ie, $U_{ij}^{hm_j}(k) \triangleq E\left\{\tilde{x}_i^h(k)\left(\tilde{z}_j^{m_j}(k)\right)'\right\}$

$U_{ji}^{m_jh}(k)$ cross covariance matrix of $z_j^{m_j}(k)$ and $\bar{x}_i^h(k)$, ie, $U_{ji}^{m_jh}(k) \triangleq E\left\{\tilde{z}_j^{m_j}(k)\left(\tilde{x}_i^h(k)\right)'\right\}$

$v(k)$ zero-mean white Gaussian target process noise at update (k)

V_m "volume" in measurement space in which a target can be

V_s "volume" in state space in which a target can be

$x_i(k)$ the state vector for target t_i (ie, its true state) at time k

$\bar{x}_i^h(k)$ state estimate/prediction of target t_i at time k resulting from the fusion of h earlier state estimates at time k and the fused estimate from time $k-1$

$\tilde{x}_i^h(k)$ error corresponding to the estimate $\bar{x}_i^h(k)$, ie, $\tilde{x}_i^h(k) \triangleq x_i(k) - \bar{x}_i^h(k)$

$y_j^{m_j}(k)$ the estimate in ground coordinates corresponding to $\psi_j(k)$ assuming propagation path m_j

$\tilde{y}_j^{m_j}(k)$ error corresponding to the estimate $y_j^{m_j}(k)$, ie, $\tilde{y}_j^{m_j}(k) \triangleq x_i(k) - y_j^{m_j}(k)$

$z_j^{m_j}(k)$ the measurement in ground coordinates corresponding to $\delta_j(k)$ assuming propagation path m_j

$\bar{z}_i^h(k)$ the measurement prediction (the expected value) for target t_i at time k resulting from the fusion of h earlier state estimates at time k and the fused estimate from time $k-1$

$\tilde{z}_j^{m_j}(k)$ error corresponding to the measurement $z_j^{m_j}(k)$, ie, $\tilde{z}_j^{m_j}(k) = Hx_i(k) - z_j^{m_j}(k)$

1 Introduction

In skywave over-the-horizon radar (OTHR), ionospheric refraction of HF signals is exploited to detect and track targets well beyond the line-of-sight horizon, thus providing surveillance over much larger areas than is possible with conventional ground based microwave radar. Unfortunately, ionospheric propagation conditions are usually such that several propagation paths exist between the target and the radar site, thus giving multiple resolved measurements for a single target. Figure 1 shows a simple model of the multiple propagation path phenomenon. The figure shows a transmitter at point O and target at point P, with two ionospheric layers, the E layer and the F layer, via which the transmitted and returned signals can travel. A signal transmitted from point O can travel via one of two paths to point P, and similarly, a signal returning from point P to point O can travel by one of two paths. If the path lengths for the four path combinations are sufficiently different for resolved returns to occur, this leads to four "apparent" targets when there is actually only one.

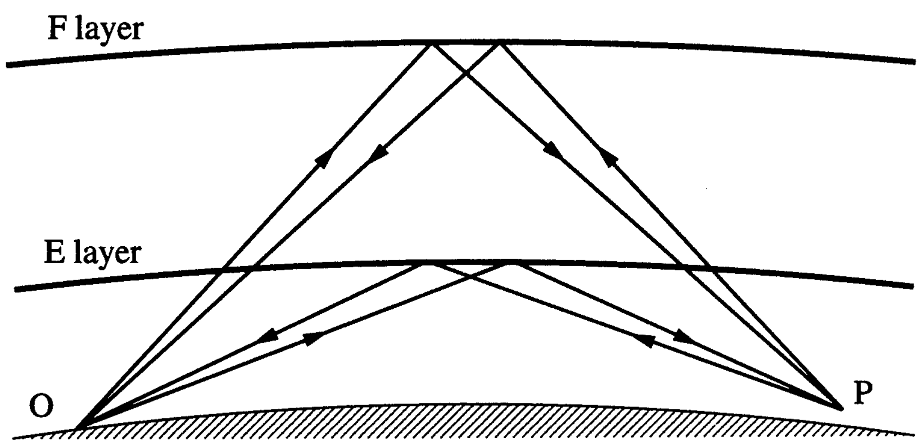

Figure 1: Multipath propagation via ionospheric E and F layers

Current practice is for OTHR tracking to be implemented in radar coordinates (*group range, group range rate, apparent azimuth* and *apparent azimuth rate*), with the major reason being to limit computational requirements. The result is that when multipath propagation conditions exist, multiple radar tracks are formed for each target. In situations where there is an unknown number of targets, the challenge is then to correctly associate radar tracks with targets and to subsequently estimate target locations in ground coordinates by fusing associated radar track data. Many plausible track-to-target associations may exist, so the association ambiguity must be resolved.

There have been several research efforts investigating the association and fusion of multipath OTHR tracks which have been reported in the literature [11], [12], [8], [28] as well as an approach which directly performs tracking of a target in ground coordinates using multipath measurements in the presence of clutter [22]. Of these, the work described in [8], [28], [22] represent early efforts sponsored by, or performed in, Surveillance Systems Division (SSD). Subsequently, a substantial effort was made in SSD to develop a

new algorithm for multipath track fusion (MPTF) by Percival and White. In the MPTF approach, all track-to-target association hypotheses are recursively constructed, the probability of each hypothesis is calculated and the fused target states within each hypothesis are evaluated. The approach is described in references [17], [18] and [19].

Although the MPTF technique was a promising approach to the problem, it suffered from the following deficiencies. The algorithm treats each track update independently, which results in fusion of track estimates without taking into account track history. Additionaly, the algorithm does not have an effective hypothesis management strategy, making it impractical in all but the most trivial circumstances. In the absence of a hypothesis management strategy, the number of association hypotheses produced by the MPTF algorithm quickly becomes too large for real time execution. Some preliminary work on overcoming the difficulties in hypothesis management, which was done by Percival and White, is described in Refs. [18] and [19]; however the approach proved ineffective for reasons that will be described later. A further, and very restrictive, limitation of the MPTF algorithm is its inability to deal with asynchronous track data. The outcome is that track data from multiple asynchronous sensors cannot be accomodated. Thus the algorithm cannot exploit synergies that exist when multiple OTHRs, microwave radars and other sensors are used in conjunction.

The use of multiple OTHRs in conjunction with microwave radar networks, as well as other track level information such as GPS reports, offers the potential of surveillance over a large region of the earth's surface, with substantially improved reliability and accuracy in target acquisition and tracking over a single OTHR. Skywave OTHR offers the advantage of radar coverage over a large area of the earth's surface as well as the ability to detect low flying targets well past the horizon, but suffers from drawbacks which include multiple resolved measurements of individual targets and generally inaccurate registration. In contrast, microwave radar offers much higher measurement accuracy and is largely free of the multiple measurement problem associated with OTHR. However a single microwave radar has a very limited coverage area in comparison with OTHR, particularly in the case of a ground based radar and low flying targets. GPS reports have the advantage of even greater accuracy, but have only limited availability and are certainly not available for unfriendly targets. Fusing the OTHR measurements with microwave radar measurements and GPS reports, where available, thus enables the advantages of each sensor to be utilized in areas of overlapping coverage, to achieve better accuracy and reduced association ambiguity. Also, a target that is temporarily in a Doppler blind-zone of one radar may be visible to another radar; hence track continuity can be enhanced by multi-sensor fusion. Furthermore, by taking advantage of spatial continuity of the ionospheric layers, information gained by fusing OTHR, microwave radar and GPS measurements in areas of overlapping coverage can be used to more accurately determine ionospheric layer heights in these areas, and by extrapolation, in surrounding areas. Hence the performance of the OTHR itself can be improved, not only where there is overlapping coverage, but also for areas nearby.

To perform fusion of OTHR, microwave radar and GPS measurements, the major task is again to correctly associate measurements, which in this case are from multiple sensors, with targets and to subsequently estimate target locations in ground coordinates by fusing associated radar (and GPS) track data to give a unified picture of the surveillance region. Because of the similarities between the tasks of fusing multipath tracks to that of fusing

multisensor tracks, it seems natural, as well as very desirable, that a single algorithm should be developed for both problems.

This report describes, a new dynamic multipath track fusion (DMPTF) algorithm which is capable of performing association and fusion of multipath OTHR tracks from multiple OTHRs as well as multiple microwave radar tracks which fall within the OTHR surveillance region. GPS reports from commercial aircraft can also be incorporated if required. The new algorithm builds on the earlier work done in SSD by Percival and White [17], [18], [19] by providing major extensions which overcome the limitations of the earlier algorithm. The DMPTF algorithm treats all the sources of track data in the same fashion, leading to a simple and elegant set of association and fusion equations. This algorithm can be used to enhance the value of the OTHR surveillance data from the Jindalee Facility Alice Springs (JFAS) and the Jindalee Over-the-horizon Radar Network (JORN) for air picture compilation, to aid the integration of OTHR into a multi-sensor air surveillance system, and thereby contribute to the command and control capabilities of the Australian Defence Force (ADF). Early trials of the algorithm have been performed on a test-bed as well as on an operational radar, giving very promising results.

The report describes the advances made to the earlier MPTF approach, the key advances being:

- an algorithm which enables *temporal as well as spatial* track behaviour to be accounted for during the association and fusion processes,
- a *robust strategy for the pruning of association hypotheses*, which enables the efficient implementation of the algorithm for a much larger number of tracks than was previously possible,
- the development of an algorithm for *dealing with changes in the number of tracks* during consecutive updates (a common phenomenon with multipath tracks),
- *equations for using measurements in place of track estimates in the association and fusion processes*, the aim of these equations being to enable the reduction of effects due to track dependencies,
- the development of an algorithm that can be used to *associate and fuse asynchronous non-OTHR tracks (eg., microwave radar or GPS) with the multipath OTHR tracks of multiple OTHRs in a single fusion process.*

In the earlier work by Percival and White, the association and fusion equations are obtained by analogy to multisensor work described in the literature. In addition to the advances summarized in the previous paragraph, a further contribution of this report is the first principles derivation of the original MPTF algorithm, which has confirmed errors in the equations derived using the analogy approach of the earlier work. Furthermore, the first principles approach is subsequently used to provide a rigorous derivation of the new dynamic algorithm as well as equations for using measurements in place of track estimates in the association and fusion processes. Another contribution of the report is a literature survey and discussion of possible approaches for dealing with track dependence.

This report is organized as follows: In Section 2, a first principles derivation of the earlier "static" MPTF algorithm is given. Section 3 discusses issues regarding the computational complexity of the MPTF algorithm and gives reasons why hypothesis pruning is necessary for the algorithm to be able to be implemented. In section 4, the development of a hypothesis pruning algorithm is described, and subsequently, in section 5, its implementation is discussed. Section 6 presents the new dynamic multipath track fusion (DMPTF) algorithm, and investigates approaches for dealing with track dependence. Section 7 then goes on to describe the results of initial performance testing and assessment. In section 8, a modified version of DMPTF is derived in which measurements are used in the association and fusion process in place of track estimates. Section 9 describes how the DMPTF algorithm can be used for fusion of tracks from multiple sensors such as multiple OTHRs or OTHRs and microwave radars. Finally, in section 10 a summary of the work and conclusions are presented as well as an outline of future work that is recommended. The reader is also referred to [24] and [25], which present some of the algorithms and results described in this report, but in substantially less detail than is given here.

2 Static Multipath Track Fusion

The static MPTF algorithm [17], [18], [19] generates association hypotheses by recursively constructing a *hypothesis tree* at each update of the tracks. Each path through the tree corresponds to a unique association of radar tracks with targets and available propagation paths. The algorithm evaluates the probability of each hypothesis and computes estimates of the target states within each hypothesis. Key points to note are that the tracks must be updated simultaneously and each update is treated independently by the MPTF algorithm. A description of the static algorithm is given in references [17], [18] and [19]; however, the earlier version was developed by analogy with other work and was not rigorously derived. There are also some mathematical errors in the earlier equations. For these reasons, and because the static algorithm forms the basis of the initial creation of the hypothesis tree in the DMPTF algorithm, it was necessary to derive the static equations rigorously from first principles. The derivation is presented in this section.

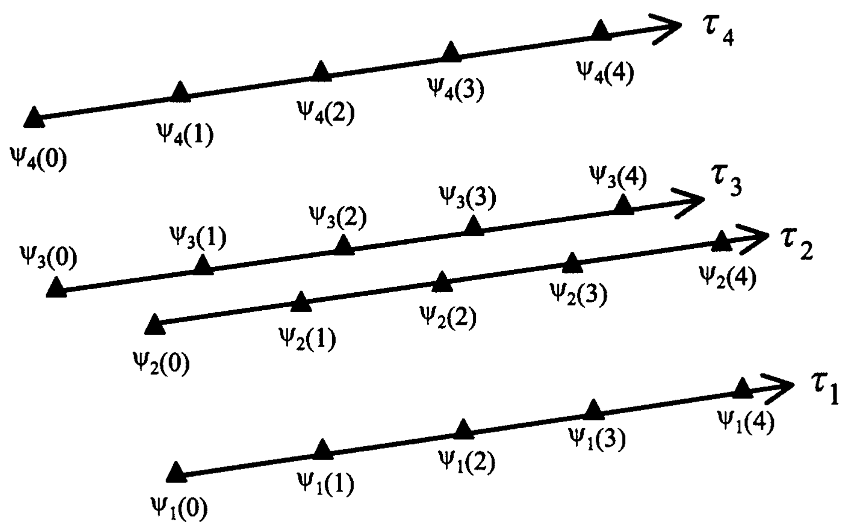

Figure 2: Four multipath tracks in radar coordinates

Consider a collection of tracks $\tau_j,\ j = 1, ..., J$ in radar coordinates. Figure 2 shows an example with four tracks (ie, $J = 4$), which often occurs with a single target. Let us first consider update (time) $k = 0$. At time $k = 0$ the J tracks are represented by the estimates $\psi_j(0),\ j = 1, ..., J$. Given prior information on the propagation path transformations from radar to ground coordinates [17], the corresponding estimates in ground coordinates $y_j^{m_j}(0)$ and their covariances $P_j^{m_j}(0)$ can be calculated for each possible propagation path m_j. Using this information, a hypothesis tree for the J tracks is created recursively, starting from the root and building the tree by considering one track estimate at a time until all the estimates have been processed. The hypothesis tree is used as a tool to visualize all the target and path associations that are possible for the J tracks. Note that for the remainder of this section the time index, (0), is not written, for the sake of brevity.

To begin, let $\lambda_{n_1 n_2 \ldots n_K}$ denote the composite hypothesis that τ_j is associated with

target n_j, $j = 1, 2, ..., K$, $K \leq J$, ie, τ_1 is associated with n_1, τ_2 is associated with n_2,..., τ_K is associated with n_K. This type of hypothesis will be referred to as a *path independent* hypothesis in the remainder of this report. As an example, figure 3 shows a path-independent hypothesis tree for a cluster of three tracks ($J = 3$). The first track is assigned to target t_1 yielding hypothesis λ_1. The second track can be due to the first target, t_1, or a second target, t_2, giving the set of hypotheses $\{\lambda_{11}, \lambda_{12}\}$. If the first two tracks were due to target t_1, the third track could be due to t_1 or could be due to a new target t_2, giving the hypotheses $\{\lambda_{111}, \lambda_{112}\}$. If the first two tracks are due to two different targets, that is t_1 and t_2 respectively, then the third track can be due to either t_1, t_2 or a new target t_3 giving the hypotheses $\{\lambda_{121}, \lambda_{122}, \lambda_{123}\}$. Thus for three tracks there are five possible path independent hypotheses $\{\lambda_{111}, \lambda_{112}, \lambda_{121}, \lambda_{122}, \lambda_{123}\}$. Note that in the general case, assuming that there are no false tracks, the values that n_j can take lie in the set $\{1, 2, ..., B_{n_1..n_{j-1}}\}$, where $B_{n_1...n_{j-1}}$ is the number of path independent hypotheses that originate from hypothesis $\lambda_{n_1...n_{j-1}}$. For example, consider λ_{12} in figure 3, where $B_{12} = 3$.

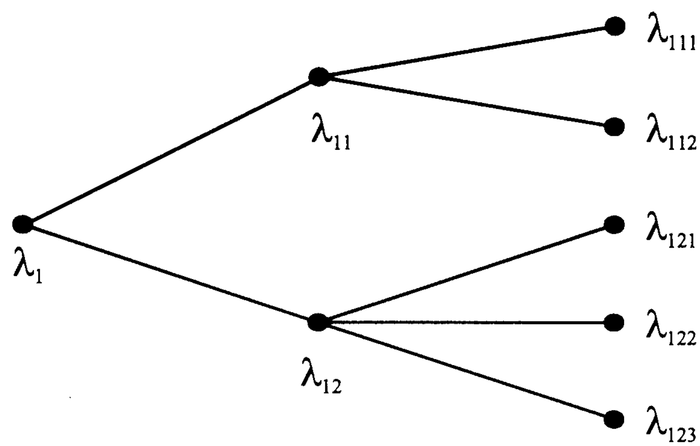

Figure 3: Path independent hypothesis tree for three tracks.

Now let $\lambda^{m_1 m_2 ... m_K}_{n_1 n_2 ... n_K}$ denote the composite hypothesis that τ_j is associated with target n_j and propagation path m_j, $j = 1, 2, ..., K$, $K \leq J$. This type of hypothesis will be referred to as a *path dependent* hypothesis. The values that can be taken by n_j are as before, and m_j lies in the set $\{1, 2, ..., M_j\}$ where M_j is the number of propagation paths associated with ψ_j. Note that M_j is known *a priori*, as it is supplied as part of the coordinate registration (CR) information. If it is assumed that no two resolved ground tracks that are due to the same target can be associated with the same propagation path, then the values that m_j can take lie in the set $\{1, 2, ..., M_j\} \cap \bar{S}_j$ where $S_j = \{m_j : m_j = m_{j-1}$ if $n_j = n_{j-1}$, $m_j = m_{j-2}$ if $n_j = n_{j-2}$,..., $m_j = m_1$ if $n_j = n_1\}$ and $\bar{S}_j$ is the complementary set of S_j. This assumption is made for the remainder of the report. Clearly, each path-independent hypothesis $\lambda_{n_1,..,n_J}$ corresponds to a set of *path-dependent* hypotheses $\{\lambda^{m_1,..,m_J}_{n_1,..,n_J}\}$. The relationship between path-dependent and path-independent hypotheses can be illustrated using the previous example of a cluster of $J = 3$ radar tracks and letting the number of propagation paths be $M_j = 2$. Consider the sequence of hypotheses, $\lambda_1 \rightarrow \lambda_{12} \rightarrow \lambda_{121}$, in

the path independent hypothesis tree of Figure 3. This sequence can be decomposed into the path dependent hypotheses shown in Figure 4.

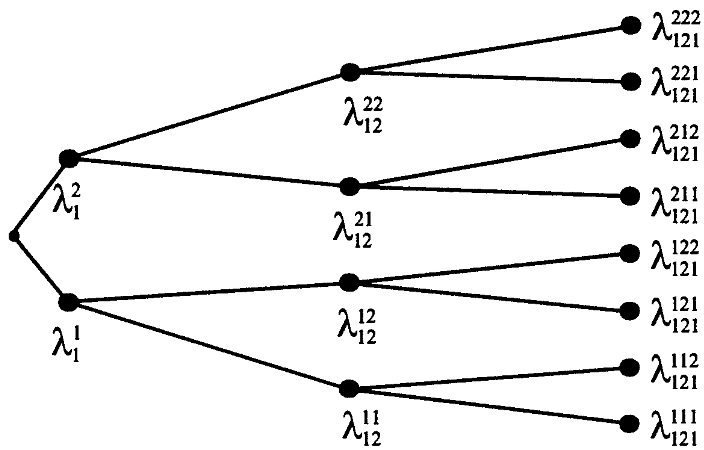

Figure 4: Path dependent hypotheses corresponding to path $\lambda_1 \rightarrow \lambda_{12} \rightarrow \lambda_{121}$ of the path independent hypothesis tree.

Consider now, in detail, the creation of the hypothesis tree and the calculation of the hypothesis probabilities and fused estimates.

Firstly, let us define the following:

$\theta_j^{m_j} \triangleq$ the event "track $\tau_j,\ j = 1,..,J$ is associated with propagation via propagation path $m_j,\ m_j \in \{1, 2, .., M_j\}$".

$\phi_j^{n_j} \triangleq$ the event that "track $\tau_j,\ j = 1,..,J$ is associated with target $n_j,\ n_j \in \{1, 2, .., J\}$".

From the first estimate, ψ_1, the propagation path transformations can be used to calculate $y_1^{m_1}$ and its covariance $P_1^{m_1}$ for each propagation path $m_1 = 1,..,M_1$. Under the assumption that there are no false tracks, there is only one target association possible for ψ_1, namely target 1. The probability of each of the path dependent hypotheses is then

$$\begin{aligned} P\{\lambda_1^{m_1}|\psi_1\} &= P\{\theta_1^{m_1}, \phi_1^1|\psi_1\} \qquad\qquad m_1 = 1,..,M_1 \\ &= P\{\theta_1^{m_1}|\phi_1^1, \psi_1\} P\{\phi_1^1|\psi_1\} \end{aligned}$$

where

$$P\{\phi_1^1|\psi_1\} = 1$$

hence

$$\begin{aligned} P\{\lambda_1^{m_1}|\psi_1\} &= P\{\theta_1^{m_1}|\phi_1^1, \psi_1\} \\ &= P\{\theta_1^{m_1}\} \end{aligned}$$

Let $\beta_j^{m_j}$ be the *prior* probability of $\theta_j^{m_j}$; this is estimated using physical measurements of the ionosphere. Hence $P\{\theta_1^{m_1}\} = \beta_1^{m_1}$, and

$$P\{\lambda_1^{m_1}|\psi_1\} = \beta_1^{m_1} \qquad m_1 = 1,..,M_1 \tag{1}$$

For each subsequent estimate, $\psi_j,\ j = 2,..,J$, the recursive equation for calculating the probability of $\lambda_{n_1..n_K}^{m_1..m_K}$ from $\lambda_{n_1..n_{K-1}}^{m_1..m_{K-1}}$ can be derived as follows.

Consider the second estimate, ψ_2; using the propagation path transformations we can then calculate $y_2^{m_2}$ and its covariance $P_2^{m_2}$ for each propagation path, ie, $m_2 = 1,..,M_2$, where M_2 is the number possible paths for τ_2. Define now ${}^j\lambda_{n_j}^{m_j} \triangleq$ the (single path dependent) hypothesis that τ_j is associated with target $n_j \in \{1, 2, .., B_{n_1..n_{j-1}}\}$ and propagation path $m_j \in \{1, 2, .., M_j\}$.

Consider now hypothesis ${}^2\lambda_{n_2}^{m_2}$, ie, that track τ_2 is associated with target n_2 and with propagation path m_2. The probability of the hypothesis is

$$
\begin{aligned}
P\left\{{}^2\lambda_{n_2}^{m_2}\left|\psi_2, \psi_1, \lambda_1^{m_1}\right.\right\} &= P\left\{\theta_2^{m_2}, \phi_2^{n_2}\left|\psi_2, \psi_1, \lambda_1^{m_1}\right.\right\} \\
&= \frac{p\left(\psi_2\left|\theta_2^{m_2}, \phi_2^{n_2}, \psi_1, \lambda_1^{m_1}\right.\right) P\left\{\theta_2^{m_2}, \phi_2^{n_2}\left|\psi_1, \lambda_1^{m_1}\right.\right\}}{p\left(\psi_2\left|\psi_1, \lambda_1^{m_1}\right.\right)} \\
&= \frac{p\left(\psi_2\left|\theta_2^{m_2}, \phi_2^{n_2}, \psi_1, \lambda_1^{m_1}\right.\right) P\left\{\theta_2^{m_2}\left|\phi_2^{n_2}, \psi_1, \lambda_1^{m_1}\right.\right\} P\left\{\phi_2^{n_2}\left|\psi_1, \lambda_1^{m_1}\right.\right\}}{\sum_{\bar{n}_2=1}^{2}\sum_{\bar{m}_2=1}^{M_2} p\left(\psi_2\left|\theta_2^{\bar{m}_2}, \phi_2^{\bar{n}_2}, \psi_1, \lambda_1^{m_1}\right.\right) P\left\{\theta_2^{\bar{m}_2}, \phi_2^{\bar{n}_2}\left|\psi_1, \lambda_1^{m_1}\right.\right\}} \\
&= \frac{p\left(\psi_2\left|\theta_2^{m_2}, \phi_2^{n_2}, \psi_1, \lambda_1^{m_1}\right.\right) P\left\{\theta_2^{m_2}\left|\phi_2^{n_2}, \psi_1, \lambda_1^{m_1}\right.\right\} P\left\{\phi_2^{n_2}\left|\psi_1, \lambda_1^{m_1}\right.\right\}}{\sum_{\bar{n}_2=1}^{2}\sum_{\bar{m}_2=1}^{M_2} p\left(\psi_2\left|\theta_2^{\bar{m}_2}, \phi_2^{\bar{n}_2}, \psi_1, \lambda_1^{m_1}\right.\right) P\left\{\theta_2^{\bar{m}_2}\left|\phi_2^{\bar{n}_2}, \psi_1, \lambda_1^{m_1}\right.\right\} P\left\{\phi_2^{\bar{n}_2}, \left|\psi_1, \lambda_1^{m_1}\right.\right\}}
\end{aligned}
$$

Now, in the above $P\left\{\theta_2^{m_2}\left|\phi_2^{n_2}, \psi_1, \lambda_1^{m_1}\right.\right\} = P\left\{\theta_2^{m_2}\right\} = \beta_2^{m_2},\ m_2 = 1,..,M_2$ because the conditions give no information regarding the probability of $\theta_2^{m_2}$. With regard to $P\left\{\phi_2^{n_2}\left|\psi_1, \lambda_1^{m_1}\right.\right\},\ n_2 = 1, 2$, there are (at least) two approaches that may be considered:

Approach 1: Assume that we know nothing about the target density, and simply assume that both ϕ_2^1 and ϕ_2^2 are equally likely, ie, $P\left\{\phi_2^1\left|\psi_1, \lambda_1^{m_1}\right.\right\} = \frac{1}{B_1} = \frac{1}{2}$ and $P\left\{\phi_2^2\left|\psi_1, \lambda_1^{m_1}\right.\right\} = \frac{1}{B_1} = \frac{1}{2}$. This is the approach that will be developed here.

Approach 2: Assume that we know something about the target density *a priori* and use this to determine $P\left\{\phi_2^{n_2}\left|\psi_1, \lambda_1^{m_1}\right.\right\}$. This approach will not be investigated in this report.

Now, in the implementation of MPTF the ground co-ordinate estimates $y_j^{m_j}$ are used to perform the probability calculations, hence we need an equation in terms of them. To obtain this, wherever we encounter the conditioning event $\theta_j^{m_j}$ we can replace ψ_j with $y_j^{m_j}$, ie, replace the estimate in radar coordinates with the equivalent estimate in ground coordinates given the stated propagation path. Hence we can easily obtain

$$
P\left\{{}^2\lambda_{n_2}^{m_2}\left|\psi_2, \psi_1, \lambda_1^{m_1}\right.\right\} = \frac{p\left(y_2^{m_2}\left|\theta_2^{m_2}, \phi_2^{n_2}, y_1^{m_1}, \lambda_1^{m_1}\right.\right) P\left\{\theta_2^{m_2}\left|\phi_2^{n_2}, y_1^{m_1}, \lambda_1^{m_1}\right.\right\} P\left\{\phi_2^{n_2}\left|y_1^{m_1}, \lambda_1^{m_1}\right.\right\}}{\sum_{\bar{n}_2=1}^{2}\sum_{\bar{m}_2=1}^{M_2} p\left(y_2^{\bar{m}_2}\left|\theta_2^{\bar{m}_2}, \phi_2^{\bar{n}_2}, y_1^{m_1}, \lambda_1^{m_1}\right.\right) P\left\{\theta_2^{\bar{m}_2}\left|\phi_2^{\bar{n}_2}, y_1^{m_1}, \lambda_1^{m_1}\right.\right\} P\left\{\phi_2^{\bar{n}_2}, \left|y_1^{m_1}, \lambda_1^{m_1}\right.\right\}}
$$

Substituting

$$\begin{aligned} P\left\{\theta_2^{m_2}\left|\phi_2^{n_2}, y_1^{m_1}, \lambda_1^{m_1}\right.\right\} &= \beta_2^{m_2} \\ P\left\{\phi_2^1\left|y_1^{m_1}, \lambda_1^{m_1}\right.\right\} &= \frac{1}{2} \\ P\left\{\phi_2^2\left|y_1^{m_1}, \lambda_1^{m_1}\right.\right\} &= \frac{1}{2} \end{aligned}$$

into the above gives

$$P\left\{{}^2\lambda_{n_2}^{m_2}\left|\psi_2, \psi_1, \lambda_1^{m_1}\right.\right\} = \frac{p\left(y_2^{m_2}\left|\theta_2^{m_2}, \phi_2^{n_2}, y_1^{m_1}, \lambda_1^{m_1}\right.\right)\beta_2^{m_2}}{\sum_{\bar{n}_2=1}^{2}\sum_{\bar{m}_2=1}^{M_2} p\left(y_2^{\bar{m}_2}\left|\theta_2^{\bar{m}_2}, \phi_2^{\bar{n}_2}, y_1^{m_1}, \lambda_1^{m_1}\right.\right)\beta_2^{\bar{m}_2}}$$

The probability of the composite hypothesis $\lambda_{1n_2}^{m_1 m_2}$ is hence given by

$$\begin{aligned} P\left\{\lambda_{1n_2}^{m_1 m_2}\left|\psi_2, \psi_1\right.\right\} &= P\left\{{}^2\lambda_{n_2}^{m_2}, \lambda_1^{m_1}\left|\psi_2, \psi_1\right.\right\} \\ &= P\left\{{}^2\lambda_{n_2}^{m_2}\left|\lambda_1^{m_1}, \psi_2, \psi_1\right.\right\} P\left\{\lambda_1^{m_1}\left|\psi_1\right.\right\} \\ &= \frac{p\left(y_2^{m_2}\left|\theta_2^{m_2}, \phi_2^{n_2}, y_1^{m_1}, \lambda_1^{m_1}\right.\right)\beta_2^{m_2}}{\sum_{\bar{n}_2=1}^{2}\sum_{\bar{m}_2=1}^{M_2} p\left(y_2^{\bar{m}_2}\left|\theta_2^{\bar{m}_2}, \phi_2^{\bar{n}_2}, y_1^{m_1}, \lambda_1^{m_1}\right.\right)\beta_2^{\bar{m}_2}}\beta_1^{m_1} \end{aligned}$$

where it has been assumed that

$$P\left\{\lambda_1^{m_1}\left|\psi_2, \psi_1\right.\right\} = P\left\{\lambda_1^{m_1}\left|\psi_1\right.\right\}$$

ie, that the probability of $\lambda_1^{m_1}$ is independent of ψ_2 prior to any hypothesis being made about the target and propagation path associated with ψ_2.

Consider now the general case of calculating the probability of $\lambda_{n_1..n_K}^{m_1..m_K}$ from $\lambda_{n_1..n_{K-1}}^{m_1..m_{K-1}}$. We then have the following

$$\begin{aligned} P\left\{{}^K\lambda_{n_K}^{m_K}\left|\psi_1, .., \psi_K, \lambda_{n_1..n_{K-1}}^{m_1..m_{K-1}}\right.\right\} &= P\left\{\theta_K^{m_K}, \phi_K^{n_K}\left|\psi_1, .., \psi_K, \lambda_{n_1..n_{K-1}}^{m_1..m_{K-1}}\right.\right\} \\ m_j &\in \{1, 2, .., M_j\},\ n_j \in \left\{1, 2, .., B_{n_1..n_{j-1}}\right\},\ j = 1, .., K \end{aligned}$$

Let us now introduce the following shorthand notation: $\Psi^K \triangleq \psi_1, .., \psi_K$, where again the time index, (0) , is omitted for brevity. Then

$$\begin{aligned} &P\left\{{}^K\lambda_{n_K}^{m_K}\left|\Psi^K, \lambda_{n_1..n_{K-1}}^{m_1..m_{K-1}}\right.\right\} = P\left\{\theta_K^{m_K}, \phi_K^{n_K}\left|\Psi^K, \lambda_{n_1..n_{K-1}}^{m_1..m_{K-1}}\right.\right\} \\ &= \frac{p\left(\psi_K\left|\theta_K^{m_K}, \phi_K^{n_K}, \Psi^{K-1}, \lambda_{n_1..n_{K-1}}^{m_1..m_{K-1}}\right.\right) P\left\{\theta_K^{m_K}, \phi_K^{n_K}\left|\Psi^{K-1}, \lambda_{n_1..n_{K-1}}^{m_1..m_{K-1}}\right.\right\}}{p\left(\psi_K\left|\Psi^{K-1}, \lambda_{n_1..n_{K-1}}^{m_1..m_{K-1}}\right.\right)} \\ &= \frac{\left[\begin{array}{c} p\left(\psi_K\left|\theta_K^{m_K}, \phi_K^{n_K}, \Psi^{K-1}, \lambda_{n_1..n_{K-1}}^{m_1..m_{K-1}}\right.\right) \times \\ P\left\{\theta_K^{m_K}\left|\phi_K^{n_K}, \Psi^{K-1}, \lambda_{n_1..n_{K-1}}^{m_1..m_{K-1}}\right.\right\} \times P\left\{\phi_K^{n_K}\left|\Psi^{K-1}, \lambda_{n_1..n_{K-1}}^{m_1..m_{K-1}}\right.\right\} \end{array}\right]}{\sum_{\bar{n}_K=1}^{B_{n_1..n_{K-1}}}\sum_{\bar{m}_K=1}^{M_K} p\left(\psi_K\left|\theta_K^{\bar{m}_K}, \phi_K^{\bar{n}_K}, \Psi^{K-1}, \lambda_{n_1..n_{K-1}}^{m_1..m_{K-1}}\right.\right) P\left\{\theta_K^{\bar{m}_K}, \phi_K^{\bar{n}_K}\left|\Psi^{K-1}, \lambda_{n_1..n_{K-1}}^{m_1..m_{K-1}}\right.\right\}} \\ &= \frac{\left[\begin{array}{c} p\left(\psi_K\left|\theta_K^{m_K}, \phi_K^{n_K}, \Psi^{K-1}, \lambda_{n_1..n_{K-1}}^{m_1..m_{K-1}}\right.\right) \times \\ P\left\{\theta_K^{m_K}\left|\phi_K^{n_K}, \Psi^{K-1}, \lambda_{n_1..n_{K-1}}^{m_1..m_{K-1}}\right.\right\} \times P\left\{\phi_K^{n_K}\left|\Psi^{K-1}, \lambda_{n_1..n_{K-1}}^{m_1..m_{K-1}}\right.\right\} \end{array}\right]}{\sum_{\bar{n}_K=1}^{B_{n_1..n_{K-1}}}\sum_{\bar{m}_K=1}^{M_K}\left[\begin{array}{c} p\left(\psi_K\left|\theta_K^{\bar{m}_K}, \phi_K^{\bar{n}_K}, \Psi^{K-1}, \lambda_{n_1..n_{K-1}}^{m_1..m_{K-1}}\right.\right) \times \\ P\left\{\theta_K^{\bar{m}_K}\left|\phi_K^{\bar{n}_K}, \Psi^{K-1}, \lambda_{n_1..n_{K-1}}^{m_1..m_{K-1}}\right.\right\} P\left\{\phi_K^{\bar{n}_K}, \left|\Psi^{K-1}, \lambda_{n_1..n_{K-1}}^{m_1..m_{K-1}}\right.\right\} \end{array}\right]} \end{aligned}$$

As stated earlier, the relevant estimates for calculating the probabilities are the ground co-ordinate estimates $y_j^{m_j}$. The equation in terms of them is

$$P\left\{{}^K\lambda_{n_K}^{m_K}\left|\Psi^K,\lambda_{n_1..n_{K-1}}^{m_1..m_{K-1}}\right.\right\} = \frac{\left[\begin{array}{c} p\left(y_K^{m_K}\left|\theta_K^{m_K},\phi_K^{n_K},y_1^{m_1},..,y_{K-1}^{m_{K-1}},\lambda_{n_1..n_{K-1}}^{m_1..m_{K-1}}\right.\right) \\ \times P\left\{\theta_K^{m_K}\left|\phi_K^{n_K},y_1^{m_1},..,y_{K-1}^{m_{K-1}},\lambda_{n_1..n_{K-1}}^{m_1..m_{K-1}}\right.\right\} \\ \times P\left\{\phi_K^{n_K}\left|y_1^{m_1},..,y_{K-1}^{m_{K-1}},\lambda_{n_1..n_{K-1}}^{m_1..m_{K-1}}\right.\right\}\end{array}\right]}{\sum_{\bar{n}_K=1}^{B_{n_1..n_{K-1}}}\sum_{\bar{m}_K=1}^{M_K}\left[\begin{array}{c} p\left(y_K^{\bar{m}_K}\left|\theta_K^{\bar{m}_K},\phi_K^{\bar{n}_K},y_1^{m_1},..,y_{K-1}^{m_{K-1}},\lambda_{n_1..n_{K-1}}^{m_1..m_{K-1}}\right.\right) \\ \times P\left\{\theta_K^{\bar{m}_K}\left|\phi_K^{\bar{n}_K},y_1^{m_1},..,y_{K-1}^{m_{K-1}},\lambda_{n_1..n_{K-1}}^{m_1..m_{K-1}}\right.\right\} \\ \times P\left\{\phi_K^{\bar{n}_K},\left|y_1^{m_1},..,y_{K-1}^{m_{K-1}},\lambda_{n_1..n_{K-1}}^{m_1..m_{K-1}}\right.\right\}\end{array}\right]}$$

Substituting

$$\begin{aligned} P\left\{\theta_K^{m_K}\left|\phi_K^{n_K},y_1^{m_1},..,y_{K-1}^{m_{K-1}},\lambda_{n_1..n_{K-1}}^{m_1..m_{K-1}}\right.\right\} &= \beta_K^{m_K} \\ P\left\{\phi_K^{n_K}\left|y_1^{m_1},..,y_{K-1}^{m_{K-1}},\lambda_{n_1..n_{K-1}}^{m_1..m_{K-1}}\right.\right\} &= \frac{1}{B_{n_1..n_{K-1}}} \end{aligned}$$

into the above gives

$$P\left\{{}^K\lambda_{n_K}^{m_K}\left|\Psi^K,\lambda_{n_1..n_{K-1}}^{m_1..m_{K-1}}\right.\right\} = \frac{p\left(y_K^{m_K}\left|\theta_K^{m_K},\phi_K^{n_K},y_1^{m_1},..,y_{K-1}^{m_{K-1}},\lambda_{n_1..n_{K-1}}^{m_1..m_{K-1}}\right.\right)\beta_K^{m_K}}{\sum_{\bar{n}_K=1}^{B_{n_1..n_{K-1}}}\sum_{\bar{m}_K=1}^{M_K} p\left(y_K^{\bar{m}_K}\left|\theta_K^{\bar{m}_K},\phi_K^{\bar{n}_K},y_1^{m_1},..,y_{K-1}^{m_{K-1}},\lambda_{n_1..n_{K-1}}^{m_1..m_{K-1}}\right.\right)\beta_K^{\bar{m}_K}}$$

The probability of the composite hypothesis $P\left\{\lambda_{n_1..n_K}^{m_1..m_K}\left|\Psi^K\right.\right\}$ is

$$P\left\{\lambda_{n_1..n_K}^{m_1..m_K}\left|\Psi^K\right.\right\} = P\left\{{}^K\lambda_{n_K}^{m_K},\lambda_{n_1..n_{K-1}}^{m_1..m_{K-1}}\left|\Psi^K\right.\right\}$$

Now, applying conditional probability and assuming that[1]

$$P\left\{\lambda_{n_1..n_{K-1}}^{m_1..m_{K-1}}\left|\Psi^K\right.\right\} = P\left\{\lambda_{n_1..n_{K-1}}^{m_1..m_{K-1}}\left|\Psi^{K-1}\right.\right\}$$

ie, that the probability of $\lambda_{n_1..n_{K-1}}^{m_1..m_{K-1}}$ is independent of ψ_K prior to any hypothesis being made about the target and propagation path associated with ψ_K, the following is obtained

$$P\left\{\lambda_{n_1..n_K}^{m_1..m_K}\left|\Psi^K\right.\right\} = P\left\{{}^K\lambda_{n_K}^{m_K}\left|\lambda_{n_1..n_{K-1}}^{m_1..m_{K-1}},\Psi^K\right.\right\}P\left\{\lambda_{n_1..n_{K-1}}^{m_1..m_{K-1}}\left|\Psi^{K-1}\right.\right\}$$

ie,

$$P\left\{\lambda_{n_1..n_K}^{m_1..m_K}\left|\Psi^K\right.\right\} = \frac{p\left(y_K^{m_K}\left|\theta_K^{m_K},\phi_K^{n_K},y_1^{m_1},..,y_{K-1}^{m_{K-1}},\lambda_{n_1..n_{K-1}}^{m_1..m_{K-1}}\right.\right)\beta_K^{m_K}P\left\{\lambda_{n_1..n_{K-1}}^{m_1..m_{K-1}}\left|\Psi^{K-1}\right.\right\}}{\sum_{\bar{n}_K=1}^{B_{n_1..n_{K-1}}}\sum_{\bar{m}_K=1}^{M_K} p\left(y_K^{\bar{m}_K}\left|\theta_K^{\bar{m}_K},\phi_K^{\bar{n}_K},y_1^{m_1},..,y_{K-1}^{m_{K-1}},\lambda_{n_1..n_{K-1}}^{m_1..m_{K-1}}\right.\right)\beta_K^{\bar{m}_K}}$$

[1]It may be possible to avoid this assumption, which has been found to introduce an ordering property on the hypothesis probabilities, that is, the magnitude of the probabilities is affected by the order in which the tracks are introduced to the algorithm. A new derivation, which aims to remove the assumption is in progress and will be presented in a subsequent publication.

If it is assumed that no two resolved ground tracks which are due to the same target can be associated with the same propagation path then

$$P\left\{\lambda_{n_1..n_K}^{m_1..m_K}\left|\Psi^K\right.\right\} = \frac{p\left(y_K^{m_K}\left|y_1^{m_1},..,y_{K-1}^{m_{K-1}},\lambda_{n_1..n_K}^{m_1..m_K}\right.\right)\beta_K^{m_K}P\left\{\lambda_{n_1..n_{K-1}}^{m_1..m_{K-1}}\left|\Psi^{K-1}\right.\right\}}{\sum\limits_{\bar{n}_K=1}^{B_{n_1..n_{K-1}}}\sum\limits_{\substack{\bar{m}_K=1,\\ \bar{m}_K\notin S_K}}^{M_K} p\left(y_K^{\bar{m}_K}\left|y_1^{m_1},..,y_{K-1}^{m_{K-1}},\lambda_{n_1..n_{K-1}\bar{n}_K}^{m_1..m_{K-1}\bar{m}_K}\right.\right)\beta_K^{\bar{m}_K}} \tag{2}$$

Now, let us consider how the likelihood $p\left(y_K^{m_K}\left|\theta_K^{m_K},\phi_K^{n_K},y_1^{m_1},..,y_{K-1}^{m_{K-1}},\lambda_{n_1..n_{K-1}}^{m_1..m_{K-1}}\right.\right)$ is calculated. If $n_K \neq n_j,\ j = 1,..,K-1$ ie, ψ_K represents a new target, we have no prior information regarding what the state of $y_K^{m_K}$ should be. Hence the likelihood, $\Lambda_K^{m_K}$, is given by:

$$\begin{aligned}\Lambda_K^{m_K} &\triangleq p\left(y_K^{m_K}\left|\theta_K^{m_K},\phi_K^{n_K},y_1^{m_1},..,y_{K-1}^{m_{K-1}},\lambda_{n_1..n_{K-1}}^{m_1..m_{K-1}}\right.\right) \\ &= \frac{1}{V_s}\end{aligned} \tag{3}$$

where V_s is the "*volume*" in state space in which the target may be. The above equation is equivalent to saying that the target is equally likely to be anywhere in the volume. Note that this "*volume*" represents all the possible values of the state vector $[r\ \dot{r}\ a\ \dot{a}]'$ where r is the range of the target in ground coordinates, and a is the azimuth of the target in ground coordinates.

Now consider the case where $n_K = n_j$ for one or more of $j = 1,..,K-1$ ie, ψ_K represents the same target as at least one previously hypothesized target. Let us assume that the hypothesis $\lambda_{n_1..n_{K-1}}^{m_1..m_{K-1}}$ is associated with T targets where $1 \leq T \leq K-1$. Assume now that the target that τ_K corresponds to is t_i, $1 \leq i \leq T$ and that prior to considering $y_K^{m_K}$, there have been h track estimates associated with the current target for this time instant. Let the previous estimate of the state of target t_i be $\bar{x}_i^h$ and its covariance $\bar{P}_i^h$. Since it is assumed that $y_K^{m_K}$ (with covariance $P_K^{m_K}$) is from target t_i , the best available estimate of $y_K^{m_K}$, prior to actually determining $y_K^{m_K}$ from τ_K and the assumed propagation path m_K , is obtained by noting that

- all process and measurement noises are assumed Gaussian,
- $y_K^{m_K}$ is computed using a Kalman filter (which is an unbiased estimator if the process and measurement models are correct) and a subsequent ionospheric transformation which is assumed unbiased,
- $\bar{x}_i^h$ is computed using an unbiased estimator,
- $y_K^{m_K}$ and $\bar{x}_i^h$ are both estimates of the same target at the same time instant.

Assuming all the above, the best estimate (ie, the expected value) of $y_K^{m_K}$ is

$$E\left\{y_K^{m_K}\left|\theta_K^{m_K},\phi_K^{n_K},y_1^{m_1},..,y_{K-1}^{m_{K-1}},\lambda_{n_1..n_{K-1}}^{m_1..m_{K-1}}\right.\right\} = \bar{x}_i^h$$

The covariance of $(y_K^{m_K} - \bar{x}_i^h)$ is

$$\begin{aligned}T_K^{m_K} &\triangleq E\left\{\left[y_K^{m_K} - \bar{x}_i^h\right]\left[y_K^{m_K} - \bar{x}_i^h\right]'\right\} \\ &= E\left\{\left[\left(y_K^{m_K} - x_i\right) - \left(\bar{x}_i^h - x_i\right)\right]\left[\left(y_K^{m_K} - x_i\right) - \left(\bar{x}_i^h - x_i\right)\right]'\right\}\end{aligned}$$

Let $\tilde{x}_i^h = x_i - \bar{x}_i^h$ and $\tilde{y}_K^{m_K} = x_i - y_K^{m_K}$, where x_i is the true state of target t_i, then

$$\begin{aligned} T_K^{m_K} &= E\left\{\left[\tilde{x}_i^h - \tilde{y}_K^{m_K}\right]\left[\tilde{x}_i^h - \tilde{y}_K^{m_K}\right]'\right\} \\ &= E\left\{\tilde{x}_i^h\left(\tilde{x}_i^h\right)' - \tilde{x}_i^h\left(\tilde{y}_K^{m_K}\right)' - \tilde{y}_K^{m_K}\left(\tilde{x}_i^h\right)' + \tilde{y}_K^{m_K}\left(\tilde{y}_K^{m_K}\right)'\right\} \\ &= E\left\{\tilde{x}_i^h\left(\tilde{x}_i^h\right)'\right\} - E\left\{\tilde{x}_i^h\left(\tilde{y}_K^{m_K}\right)'\right\} - E\left\{\tilde{y}_K^{m_K}\left(\tilde{x}_i^h\right)'\right\} + E\left\{\tilde{y}_K^{m_K}\left(\tilde{y}_K^{m_K}\right)'\right\} \end{aligned}$$

Now, $\bar{P}_i^h \triangleq E\left\{\tilde{x}_i^h\left(\tilde{x}_i^h\right)'\right\}$, $P_K^{m_K} \triangleq E\left\{\tilde{y}_K^{m_K}\left(\tilde{y}_K^{m_K}\right)'\right\}$, $P_{iK}^{hm_K} \triangleq E\left\{\tilde{x}_i^h\left(\tilde{y}_K^{m_K}\right)'\right\}$, $P_{Ki}^{m_K h} \triangleq E\left\{\tilde{y}_K^{m_K}\left(\tilde{x}_i^h\right)'\right\}$, hence

$$T_K^{m_K} = \bar{P}_i^h - P_{iK}^{hm_K} - P_{Ki}^{m_K h} + P_K^{m_K} \tag{4}$$

Then the likelihood

$$\begin{aligned} \Lambda_K^{m_K} &\triangleq p\left(y_K^{m_K}\left|\theta_K^{m_K}, \phi_K^{n_K}, y_1^{m_1}, .., y_{K-1}^{m_{K-1}}, \lambda_{n_1..n_{K-1}}^{m_1..m_{K-1}}\right.\right) \\ &= \mathcal{N}\left(y_K^{m_K}; \bar{x}_i^h, T_K^{m_K}\right) \\ &= \left|2\pi T_K^{m_K}\right|^{-\frac{1}{2}} \exp\left\{-\frac{1}{2}\left(\nu_K^{m_K}\right)'\left(T_K^{m_K}\right)^{-1}\nu_K^{m_K}\right\} \end{aligned} \tag{5}$$

where

$$\nu_K^{m_K} \triangleq \left(y_K^{m_K} - \bar{x}_i^h\right)$$

As mentioned earlier, there are some errors in some of the static equations presented in [17] and [19]. The key equations in question are the counterparts of equations 2, 3, and 5 in this report. For a description of these errors, the reader is referred to Appendix A.

Consider now the estimate $\bar{x}_i^{h+1}$ of x_i and its covariance $\bar{P}_i^{h+1}$ obtained by the fusion of $\bar{x}_i^h$, $h = 1, 2, .., H-1$ with $y_K^{m_K}$. To derive this we shall use the fundamental equations of linear estimation as shown on pages 44 and 125 of [4], ie,

$$\begin{aligned} \hat{x} &\triangleq E\{x|z\} = \bar{x} + P_{xz}P_{zz}^{-1}(z - \bar{z}) \\ P_{xx|z} &\triangleq E\left\{(x - \hat{x})(x - \hat{x})'|z\right\} = P_{xx} - P_{xz}P_{zz}^{-1}P_{zx} \end{aligned} \tag{6}$$

where x is the random vector to be estimated, z is the new measurement or the observation, $\bar{x}$ is the previous estimate of x, $\bar{z}$ is the previous estimate/prediction of z, $\hat{x} \triangleq E[x|z]$ is the conditional mean of x given z, and

$$\begin{aligned} P_{xx} &\triangleq E\left[(x - \bar{x})(x - \bar{x})'\right] \\ P_{xz} &\triangleq E\left[(x - \bar{x})(z - \bar{z})'\right] \\ P_{zx} &\triangleq E\left[(z - \bar{z})(x - \bar{x})'\right] \\ P_{zz} &\triangleq E\left[(z - \bar{z})(z - \bar{z})'\right] \\ P_{xx|z} &\triangleq E\left[(x - \hat{x})(x - \hat{x})'|z\right] \end{aligned}$$

Now, following the approach shown in sections 8.3.3 and 8.4.4 of [5], we replace the terms in the fundamental equations of linear estimation with their equivalent terms in our problem as follows

$$\begin{aligned}
\hat{x} &\rightarrow \bar{x}_i^{h+1} \\
\bar{x} &\rightarrow \bar{x}_i^{h} \\
z &\rightarrow y_K^{m_K} \\
\bar{z} &\rightarrow \bar{x}_i^{h}
\end{aligned}$$

$$P_{xz} \rightarrow E\left\{\left[x_i - \bar{x}_i^h\right]\left[y_K^{m_K} - \bar{x}_i^h\right]'\right\}$$

$$P_{zx} \rightarrow E\left\{\left[y_K^{m_K} - \bar{x}_i^h\right]\left[x_i - \bar{x}_i^h\right]'\right\}$$

$$P_{zz} \rightarrow E\left\{\left[y_K^{m_K} - \bar{x}_i^h\right]\left[y_K^{m_K} - \bar{x}_i^h\right]'\right\} = \bar{P}_i^h - P_{iK}^{hm_K} - P_{Ki}^{m_K h} + P_K^{m_K}$$

Expanding the right hand side of the replacement for P_{xz} gives

$$\begin{aligned}
E\left\{\left[x_i - \bar{x}_i^h\right]\left[y_K^{m_K} - \bar{x}_i^h\right]'\right\} &= E\left\{\left[x_i - \bar{x}_i^h\right]\left[y_K^{m_K} - x_i - \bar{x}_i^h + x_i\right]'\right\} \\
&= E\left\{\tilde{x}_i^h\left(\tilde{x}_i^h - \tilde{y}_K^{m_K}\right)'\right\} \\
&= E\left\{\tilde{x}_i^h\left(\tilde{x}_i^h\right)' - \tilde{x}_i^h\left(\tilde{y}_K^{m_K}\right)'\right\} \\
&= \bar{P}_i^h - P_{iK}^{hm_K}
\end{aligned}$$

Hence the replacement for P_{xz} is

$$P_{xz} \rightarrow \bar{P}_i^h - P_{iK}^{hm_K}$$

Expanding the right hand side of the replacement for P_{zx} gives

$$\begin{aligned}
E\left\{\left[y_K^{m_K} - \bar{x}_i^h\right]\left[x_i - \bar{x}_i^h\right]'\right\} &= E\left\{\left[y_K^{m_K} - x_i - \bar{x}_i^h + x_i\right]\left[x_i - \bar{x}_i^h\right]'\right\} \\
&= E\left\{\left(\tilde{x}_i^h - \tilde{y}_K^{m_K}\right)\left(\tilde{x}_i^h\right)'\right\} \\
&= E\left\{\tilde{x}_i^h\left(\tilde{x}_i^h\right)' - \tilde{y}_K^{m_K}\left(\tilde{x}_i^h\right)'\right\} \\
&= \bar{P}_i^h - P_{Ki}^{m_K h}
\end{aligned}$$

and hence the replacement for P_{zx} is

$$P_{zx} \rightarrow \bar{P}_i^h - P_{Ki}^{m_K h}$$

The resulting fusion equations are hence

$$\bar{x}_i^{h+1} = \bar{x}_i^h + \left(\bar{P}_i^h - P_{iK}^{hm_K}\right)\left(T_K^{m_K}\right)^{-1}\left(y_K^{m_K} - \bar{x}_i^h\right) \tag{7}$$

$$\bar{P}_i^{h+1} = \bar{P}_i^h - \left(\bar{P}_i^h - P_{iK}^{hm_K}\right)\left(T_K^{m_K}\right)^{-1}\left(\bar{P}_i^h - P_{Ki}^{m_K h}\right) \tag{8}$$

Note that the cross-covariance matrices $P_{iK}^{hm_K}$ and $P_{Ki}^{m_K h}$ are extremely difficult, if not impossible, to determine. It seems that it may be more appropriate to use the actual measurements from which the (radar coordinate) tracks are derived, than attempt to rigorously compute the cross-covariances of the track estimates. Using measurements would remove some of the contributions to the cross-covariance terms (this will be discussed later in the report). As a consequence, the approach that is taken here is to approximate the values of the cross-covariance matrices. Later, an algorithm for the creation of the hypothesis tree *using the measurements* will be derived, the aim being to compare the comparative performance of the two approaches.

As described above, the static MPTF algorithm computes the association probabilities and performs fusion for every possible combination of target and propagation path association. Unfortunately, for the numbers of tracks and propagation paths found in practice, the algorithm cannot be implemented without some form of hypothesis pruning. The same applies to the DMPTF algorithm as it uses the "static" MPTF algorithm for its initial creation of its hypothesis tree. An efficient algorithm for performing recursive pruning of the MPTF hypothesis tree has been developed and is described in section 4, following a discussion regarding the computational complexity of MPTF in section 3.

3 Computational Complexity of Multipath Track Fusion

In the absence of a hypothesis management strategy, the number of association hypotheses produced by the MPTF algorithm can quickly become too large for real time execution. Some preliminary work on overcoming this difficulty was described in Refs. [18] and [19], but the approach presented was found to be of very limited usefulness. This section presents equations for the number of hypotheses that the MPTF algorithm generates and then gives a summary of the pruning work described in [18] and [19], followed by reasons for the inadequacy of the approach.

Consider the path-independent hypothesis tree for a cluster of J tracks. The number of track-to-target association hypotheses at a given depth J of the hypothesis tree for the path independent case was evaluated in [17] to be the *Bell number* (or *exponential number*) [6]

$$H(J) = \sum_{T=1}^{J} S(J,T)$$

where $S(J,T)$ is the *Stirling number of the second kind* and is given by

$$S(J,T) = \frac{1}{T!}\sum_{t=1}^{T} (-1)^{T-t} C_t^T t^J \quad 1 \le T \le J$$

where

$$C_t^T = \frac{T!}{(T-t)!t!}.$$

Now consider the propagation paths that are to be associated with radar tracks. If all combinations of ionospheric paths are allowed, it can be easily deduced that the number of association hypotheses at a given depth J of the path dependent hypothesis tree, with M available ionospheric paths, is

$$H(J,M) = M^J \sum_{T=1}^{J} S(J,T).$$

As an example of what can be expected in real surveillance scenarios, for a cluster of 10 tracks and 9 possible ionospheric paths the number of path-dependent hypotheses in the final (i.e., 10th) level of the hypothesis tree is $H(10,9) \approx 4.04 \times 10^{14}$. The total number of hypotheses that would need to be evaluated to build the tree is

$$\bar{H}(J,M) = \sum_{j=1}^{J} H(j,M).$$

For 10 tracks and 9 possible ionospheric paths $\bar{H}(J,M) \approx 4.13 \times 10^{14}$. Clearly the hypotheses cannot be exhaustively evaluated in real time nor stored in computer memory.

An approach to reduce the number of association hypotheses *a priori* was described in Refs. [18] and [19]. In that scheme, the formulation of hypotheses relied on the application of physical (deterministic) constraints.

The assumption that the physical constraints are based on is that two radar tracks having different range estimates may arise from the same target only if the track at greatest range is associated with the propagation path of greatest path length. This prior information can be used for the *a priori* pruning of hypotheses.

The physical constraint is applied as follows. Tracks τ_j , $j = 1,..,J$ are ordered with respect to one of the components of the track state vector ψ_j, typically the range component R_j which gives

$$R_1 < R_2 < \ldots < R_J. \tag{9}$$

Likewise, the propagation paths m_j , for the jth track $j = 1,..,J$ are ordered with respect to resulting ground range $r_j^{m_j}$ so that

$$r_j^{m_j^1} < r_j^{m_j^2} < \ldots < r_j^{m_j^{M_j}}. \tag{10}$$

For path-dependent hypotheses of the form $\lambda_{n_1 n_2..n_J}^{m_1 m_2..m_J}$ where $n_i = n_j$, $i < j$, the ordering of the tracks and propagation paths in conjunction with the above assumption leads to the constraint

$$m_i < m_j. \tag{11}$$

This approach promises the advantage of decreasing the number of hypotheses that must be evaluated, hence minimizing computational expenditure. Unfortunately, as demonstrated below, at best, this is only a partial solution, as it does not prune enough hypotheses to make the MPTF algorithm practical for OTHR applications.

When the deterministic constraint is applied with J tracks and with M available ionospheric propagation paths, the number of path-dependent association hypotheses is given by

$$\tilde{H}(J, M) = \sum_{j=\max(0,J-M)}^{J-1} C_j^{J-1} C_{J-j}^{M}\, \tilde{H}(j, M),$$

where the recursive expression is initialised with $\tilde{H}(0, M) = 1$. A table giving values of $\tilde{H}(J, M)$ evaluated for $1 \leq J \leq 6$ and $1 \leq M \leq 8$ has been presented in Ref. [18]. Returning to the earlier example, with the number of tracks $J = 10$, the number of ionospheric propagation paths $M = 9$, and the constraint applied, the final level of the hypothesis tree has $\tilde{H}(10, 9) = 4.89 \times 10^{12}$ hypotheses. The total number of hypotheses is $\tilde{H} = 5.08 \times 10^{12}$, which is approximately 100 times less than the number of hypotheses in the absence of any *a priori* pruning. However, it is clear that the application of the deterministic constraint alone does not reduce the number of hypotheses anywhere near enough to make the MPTF algorithm practical. Additional hypothesis pruning methods are required regardless of whether deterministic constraints are applied. There are also other difficulties with using physical constraints which make the approach impractical even for preliminary pruning. These are described below.

Because the ordering of tracks and propagation paths is performed using *estimates* of random variables, applying the constraint (11) can give rise to errors. Firstly, for two associated radar tracks arising from propagation via the "mixed paths" such as EF and FE, the track range is nominally identical. Thus the range orderings represented by (9) and (10) may result in a correct path-dependent hypothesis not being formulated when (11) is applied. A simple method to overcome this difficulty may be formulated whereby a parameter δR which is a function of the track range variance is used to apply the constraint (11) when

$$R_i + \delta R_i < R_j - \delta R_j \qquad i < j\,. \tag{12}$$

However, this reduces the already limited amount of pruning that is achieved.

Secondly, the software implementation of the physical constraint is found to be complicated when the coordinate registration advice is considered, eg.,

- The CR system may advise a different set of propagation paths for different tracks in which case the ordering of paths according to a given track may not be appropriate to the paths advised for another track. Note that in (9) the propagation paths are ordered according to the transformations for the j^{th} track only.
- When the number of tracks and propagation paths and their order changes with time, the changing order must be somehow accommodated in formulating the updated hypotheses.

This introduces unwarranted complexities in computation and management of data as well as coding complexities. It is important to note that the original aim of *a priori* hypothesis pruning is to *reduce* computation and data management when formulating and evaluating hypotheses.

Finally, from a mathematical standpoint the physical constraint approach is essentially a *deterministic* tack-on to a stochastic algorithm (MPTF). This may have been justifiable had it achieved substantial computational efficiencies; however, this was found to not be the case.

For these reasons it was decided to abandon the physical constraint approach summarized above in favour of the recursive *a posteriori* pruning algorithm described in the following sections. Note that a simpler constraint, $m_i \neq m_j$ if $n_i = n_j$, $i \neq j$ has been retained where appropriate. In words, the constraint states that two tracks due to the same target cannot be propagating via the same propagation path. This constraint does not suffer from the disadvantages of (11).

4 Recursive Hypothesis Pruning

In order to make MPTF computationally feasible, a new pruning algorithm has been developed which exploits the recursive nature of the MPTF algorithm by pruning unlikely hypotheses at each level of the hypothesis tree as it is built, prior to creation of the next level. The decision as to which hypotheses to prune is made on the basis of hypothesis probabilities which are calculated using all components of the track estimate state vectors and path transformations. If H is the number of hypotheses formulated by the MPTF algorithm at a particular level of the hypothesis tree, then the pruning scheme retains the K best (highest probability) hypotheses. These hypotheses are subsequently used in the MPTF algorithm to form the next level of the hypothesis tree. The remaining $H - K$ hypotheses are pruned and therefore are not used in forming hypotheses in higher levels of the tree. The integration of the pruning scheme with the MPTF algorithm is outlined in the following pseudo-code, with the last two steps being the additional steps required to perform hypothesis pruning.

- *Using the first track estimate, formulate all path-dependent hypotheses and calculate their probabilities;*
- *While unprocessed track estimates remain:*
  - *Select the next track estimate;*
  - *Formulate new path-dependent hypotheses using the current track estimate and the path-dependent hypotheses remaining in the previous level of the hypothesis tree;*
  - *Calculate probabilities of all new path-dependent hypotheses;*
  - *Determine and mark the K most probable hypotheses;*
  - *Remove all unmarked hypotheses from the current level of the hypothesis tree;*

The major advantages of the new approach are:

1. It enables a huge reduction in the number of hypotheses that are evaluated, and easy tailoring to the computational capabilities of the computer on which the algorithm is to be implemented by adjusting the value of K.

2. The approach is consistent with the probabilistic model being used to model the system, using all the components of the state vector and path transformations, and their covariances.

The approach has a potential disadvantage, i.e., finding the K best out of an initial H hypotheses is computationally expensive when both K and H are large if a simple sequential search algorithm is used. In order to overcome this potential limitation, several algorithms for finding the K best hypotheses were investigated with the aim of producing a computationally efficient pruning algorithm. The following two subsections give an outline of the algorithms that were investigated and the results of comparative testing, respectively.

4.1 K-best Search Algorithms

Prior to describing the algorithms that were investigated, it is appropriate to make a distinction between hypothesis management in multiple-hypothesis tracking (MHT) [23]

and that required in MPTF. Techniques such as Lagrangian Relaxation [20], which have been applied for finding the best set of association hypotheses in MHT, are not applicable for MPTF. This is because MPTF is not cast in the form of an assignment problem, as is the case for MHT. In the assignment problem there is competition for "resources"; for example, in MHT, tracks compete for measurements. In the case of MPTF, there is no such competition. As a result, an alternative algorithm, to essentially solve a different problem, needed to be found. The development of this algorithm is outlined below.

Data structures and algorithms from the computer science literature were investigated to yield schemes that would *efficiently* find the K hypotheses with highest probability from a set of H initial hypotheses (ie, perform a *K-best Search*). Several of the most promising techniques were examined in detail and compared, with the aim of producing a computationally efficient pruning scheme to be implemented within the MPTF algorithm. The algorithms and data structures that were investigated and tested in detail were FIND [9], AVL Trees [1], [2], Skip Lists [21] and Binary Heaps [26], [10]. FIND is generally used in sorting, whereas AVL Trees and Skip Lists are generally used for searching algorithms, and the Binary Heap for constructing Priority Queues [26], [10]. In addition to the above techniques, the performance of a repeated Sequential Search was measured. This was used as a reference against which the more sophisticated approaches could be compared.

The algorithms that were examined in detail are outlined in the following paragraphs. In the algorithm descriptions, conventional computer science terminology is employed when appropriate. Specifically, a *record* is defined as an object which stores information. A *key* is a component of a record which can be used to reference the record. In a structure that has many records, their keys can be used to find individual records (ie *search*) or to order the records (ie *sort*). In the MPTF pruning algorithm the records correspond to the hypotheses and the keys are their probabilities.

Sequential Search The simplest algorithm for finding the best K of H records is to perform a Sequential Search K times. At each pass, the best record is found and then tagged so that it is not considered in subsequent searches. The algorithm is simple but inefficient when H and K are large, having a computational complexity of $O(KH)$. A summary of the algorithm follows:

Starting with a list of H records, repeat the following K times:
- *Scan through the list of records to find the record with greatest key;*
- *Record and tag that record so it is not scanned in subsequent passes;*

FIND Sorting techniques based upon Hoare's FIND algorithm [9] are widely used, for example in finding the median of a set of numbers. Consider an array $A(1), A(2), \ldots, A(K), \ldots, A(H)$ containing the keys of H records. FIND partitions the data such that $A(1)\ldots A(K-1) \geq A(K)$ and $A(K+1)\ldots A(H) \leq A(K)$, but does not require either partition to be fully ordered.

AVL Tree There are many binary tree algorithms which seek to ensure that an efficient search tree is always maintained by *balancing* the tree [26]. An example of a balanced-tree algorithm is the AVL tree developed by Adelson-Velskii and Landis [1], [2]. The AVL tree is a binary tree which has the property that the sub-trees of every node differ in height by at most one. A balance factor is added to each node in the tree and rotations of nodes within the tree, based on the balancing factor, are performed to retain a

balanced tree. While balanced-tree algorithms ensure good performance, they are complex to code and the overheads in maintaining a balanced tree are significant.

Skip List A Skip List is an ordered list data structure with additional forward pointers to skip multiple records in the list, thereby reducing the time taken to traverse the list [21]. Each record is represented by a node in the skip list. A node has *level* i if it has i forward pointers. A random number generator is used to specify the level of each node according to the skip list parameter p, which is the probability that a node has a level i pointer given that it has a level $i-1$ pointer. The parameter p is chosen to maximise algorithm performance, often set to 1/4 or 1/2. The average performance of the Skip List is in the order of $K \ln H$, but its performance reduces to that of the repeated Sequential Search in the worst case. The poor worst case performance is considered a serious limitation for real-time applications.

Binary Heap A Binary Heap [26] is a special type of binary tree, represented as an array, which maintains the first element in the array as the record with the minimum (or maximum) key. The structure is arranged such that the children of node j in the array are in position $2j$ and $2j+1$, and the parent of the jth node is in position j div 2. The elements in the structure satisfy the Heap property ie, the children of node j have larger (or smaller) keys than the record at node j. The Binary Heap is often used as a Priority Queue algorithm [10], [26] because it allows very simple and efficient implementation of the Priority Queue insertion, deletion and replacement operations.

In this work, three Priority Queue operations, *insert*, *delete* and *replace*, are supported by the Binary Heap in order to implement the K-best Search algorithm. The *insert* operation places a new record into the Heap after the previous last record, and then reorders the records so that they again satisfy the Heap property. The *delete* operation removes the top-most (i.e. the minimum key and first index in the array) record from the heap, and then reorders the remaining records as necessary. The *replace* operation replaces the top-most record in the heap with the new record and then reorders the remaining records. A detailed description of the three operations is given in Ref. [26].

The repeated Sequential Search and FIND were implemented by entering all H records into a one-dimensional array and then determining the highest K records (ie, the K records with the highest keys). With AVL Trees, Skip Lists and Binary Heaps there were two options available, owing to the data structures being used. Because the three data structures incorporate an ordering mechanism by virtue of their construction, the act of inserting a record into each structure ensures that records within the structure are ordered by their keys. AVL Trees and Skip Lists are fully ordered structures whereas Binary Heaps are only partially ordered. Most importantly, the minimum record in the structure can easily be accessed and replaced. Hence, for these three cases, two types of K-best Search algorithm were implemented. The first type sequentially inserts all H records into the structure (as was done for Sequential Search and FIND), then extracts the highest K records from the structure one-by-one. The second approach stores only K records in the structure at any one time, by monitoring the minimum record in the structure and only inserting those records that have a key which exceeds that minimum. The second algorithm is summarised by the following pseudo-code:

For the first K records in the list of H records:
- Add the records into the data structure.
For all subsequent records:
If the key of the selected record exceeds that of the minimum record already in the structure, then
- Remove the minimum key record from the structure.
- Add the new record into the structure.

A key point to note about the second approach is that it reduces the number of insertions into the structure as well as minimizing memory requirements.

4.2 Performance Comparison

The average performance of Sequential Search, FIND, AVL Tree, Skip List and Binary Heap schemes, when used as K-best Search algorithms, was compared by implementation (in C++) and testing on a 233 MHz Pentium PC, running Windows NT 4.0. Input data was simulated by generating 1000 sets of H random numbers, with the aim of determining the K highest numbers in each set. The measure of performance was the *average run time* for executing the algorithms, which was determined as a function of K and H.

Figure 5 presents the average times taken to find the K highest values from a set of H numbers. Each set of numbers was randomly generated and uniformly distributed in the interval $(0 - 1)$. The three algorithms based on ordered data structures, *(i)* Binary Heap, *(ii)* AVL tree and *(iii)* Skip List, were each implemented with structure sizes of both H and K records. In the figure, as well as the following text, the structure size is written in parentheses following the structure name.

Figure 5(a) and (c) show that the performances of the AVL Tree (H) and Skip List (H) algorithms are similar to that of Sequential search when K is small but are substantially better for $K = 500$. The fastest algorithms for most values of K and H are FIND, Binary Heap (K), AVL Tree (K), and Skip List (K). Consistently high performance is attained by the FIND and Binary Heap (K) algorithms, especially for $K = 500$, making them the best candidates for use in the hypothesis pruning algorithm. Of the two, Binary Heap (K) is considered a better choice because it requires an array of size K, in contrast to the FIND algorithm which requires an array of size H. It should also be noted that the Binary Heap (K) algorithm is simple to code, which is an important consideration in the implementation of the multipath track fusion algorithm.

For the reasons given above, the Binary Heap (K) algorithm was selected for the implementation of hypothesis pruning in MPTF.

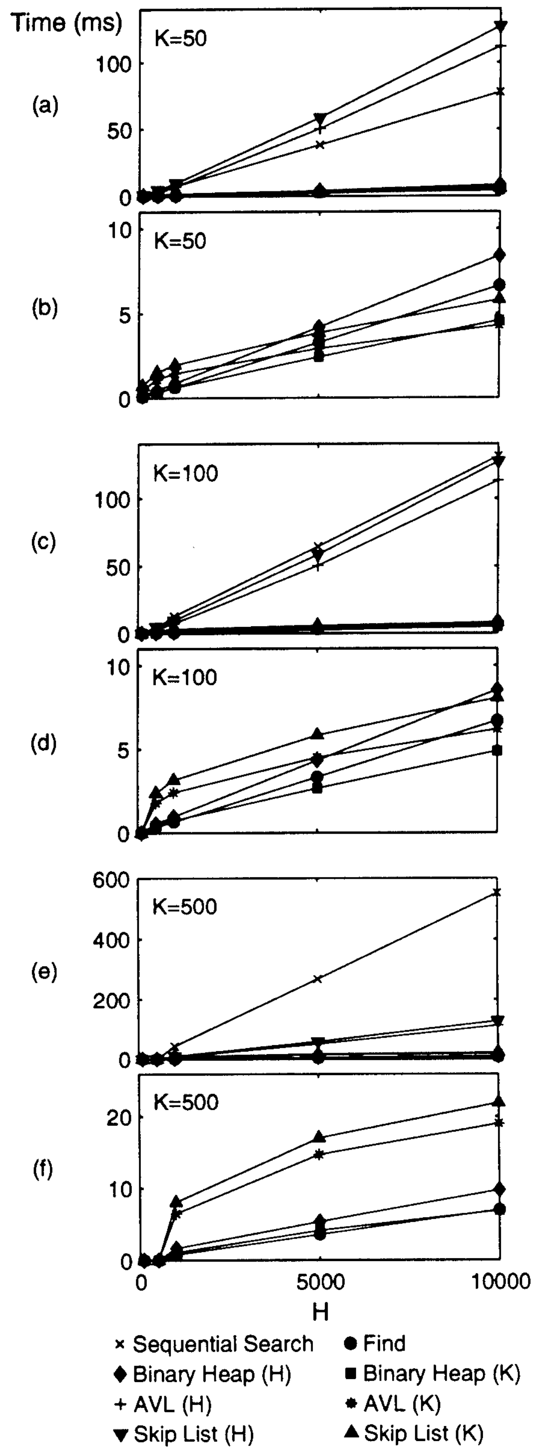

Figure 5: Comparison of average run-time *of candidate K-best Search schemes as a function of the initial number of records, H, for the cases when* $K = 50$, $K = 100$ *and* $K = 500$. *Plots (a), (c), and (e) show the same data as (b), (d) and (f), respectively, but with different scales on the vertical axes.*

5 Hypothesis Pruning Algorithm Implementation

As stated in the previous section the pruning algorithm that was found to be most efficient was the one based on a Binary Heap structure. This algorithm was coded on the MPTF testbed and its implementation was found to extend the capability of MPTF to much larger numbers of tracks and propagation paths than previously possible, making it feasible to use MPTF without further extensions when there is a moderate number of tracks and propagation paths, ie, more than 20 tracks with 10 propagation paths with computational power presently available to the MPTF algorithm in the JFAS radar. While the OTHR can be expected to have considerably more than 20 tracks over its entire coverage area, only tracks which are relatively close to one another can conceivably be due to the same target, the multiple tracks (from a single target) forming clusters of almost invariably less than 9 tracks and usually 4 tracks or less. This characteristic was used to enable MPTF to deal with very large numbers of tracks (eg., hundreds of tracks with ten propagation paths). This was achieved by the development of a clustering algorithm which groups tracks which can *conceivably* be due to the same target and then passes on these groups of tracks to the MPTF algorithm for processing. The purpose of the clustering algorithm is not to decide which tracks are due to the same target, but instead, to form groups of tracks which may be due to the same target, while maintaining group size to less than about say 20 tracks, and at the same time not placing tracks from the same target into separate clusters. The combination of pruning and clustering was implemented on the testbed and also a prototype version was incorporated in the actual Jindalee OTHR. The combination worked very well, handling tracks in the entire coverage area of the radar effectively and without creating unacceptable delays in computation. No further details of the clustering algorithm will be given here as it will be the subject of a future report.

As mentioned in an earlier section, the pruning technique that has been developed exploits the recursive nature of the MPTF algorithm by pruning low probability hypotheses at each recursion, thus avoiding the exponential growth in the number of hypotheses. A possible disadvantage of this approach is that hypotheses that are unlikely in early recursions may, under some circumstances, subsequently lead to the creation of likely hypotheses, had pruning not been performed. Preliminary investigations have indicated that this is not likely to be a problem and that the MPTF algorithm will perform very effectively with quite heavy pruning implemented in the manner described. While no detailed studies have yet been performed to determine the best choice for the number of hypotheses that should be retained, in the investigations to date, retention of the best 100 hypotheses in each algorithm recursion has proven quite successful.

Finally, it should be noted that the hypothesis pruning algorithm that has been developed here for OTHR multipath track fusion may also be applied to *multisensor* track fusion algorithms that have been cast in the multihypothesis framework.

6 Dynamic Multipath Track Fusion Algorithm

In earlier work, as described in [17] and [19], the procedure for update $k = 0$ is simply repeated for all subsequent updates ($k = 1, 2, 3, ...$), with each update being treated independently. The major disadvantages of this approach are

- only synchronous estimates can be included in the hypothesis tree, hence multisensor tracks cannot be accommodated,
- of more fundamental theoretical importance, *temporal behaviour of tracks is completely ignored*,
- when multipath tracks drop out or come into existence there is no theoretically justifiable way to "link" hypotheses from consecutive updates.

In [19], possible extensions to include target dynamics are outlined. It is suggested that the tracker's target dynamic model be used to perform predictions (in radar coordinates), which would then be transformed to ground coordinates. The extensions only involve synchronous updating of the tracks that are used as inputs to the MPTF algorithm. Of considerably more importance, there is no description of how hypothesis probability updating can be performed, other than the presentation of two very general equations that have been transcribed from [14]. These equations offer little insight into how hypothesis probability updating would actually be performed. Initiation and termination of tracks is also discussed but no solution presented.

An algorithm which overcomes all of the above-mentioned limitations is derived and presented in detail in the following subsections.

6.1 Incorporation of Target Dynamics

The dynamic multipath track fusion (DMPTF) algorithm which will now be derived overcomes the major limitations of the static MPTF algorithm by using an approach in which target behaviour is modelled. This enables hypothesis probability calculation and fusion of estimates that are temporally separated, enabling asynchronicity to be accommodated and temporal behaviour to be taken into account. In the DMPTF algorithm, the hypothesis tree constructed at $k = 0$ is as described earlier in section 2. However, instead of creating a new tree at each update, as is done in the static MPTF algorithm, hypothesis probabilities and fused estimates of the initial tree are updated as new data become available at updates $k > 0$.

The following assumptions are made when developing the algorithm:

1. If track τ_j is due to target t_i with propagation via path m_j at update $k = 0$, then it remains so for all future updates. This is clearly true if the tracking algorithm has performed its function correctly.
2. The number of tracks is constant for all updates. *This is only a temporary assumption* that is utilized in the initial development of the DMPTF algorithm and is later removed by the development of an adjunct algorithm which will be described later.

As a result, no new hypotheses are generated after the initial creation of the tree at $k = 0$. For updates $k > 0$, only the hypothesis probabilities and their associated fused estimates change.

Now let us consider updates $k = 1, 2, 3, \ldots$. The hypothesis tree built up at $k = 0$ can be used as the basis for the tree at subsequent updates as described below.

In order to shorten notation, let us define the symbol $\Psi^j(k)$ such that $\Psi^j(k) \triangleq \psi_1(k), .., \psi_j(k)$. In the creation of the hypothesis tree (at update $k = 0$) we derived the probability $P\left\{\lambda_{n_1..n_J}^{m_1..m_J} \left| \Psi^J(0)\right.\right\}$ of each path dependent hypothesis $\lambda_{n_1..n_J}^{m_1..m_J}$, $m_j \in \{1, 2, .., M_j\}$, $n_j \in \left\{1, 2, .., B_{n_1..n_{j-1}}\right\}$ given all the data available at update $k = 0$, ie the estimates $\psi_1(0), .., \psi_J(0)$, as described in section 2. Now let us define the following shorthand notation:

$$\begin{aligned} D^k &\triangleq \psi_1(0), .., \psi_J(0), \psi_1(1), .., \psi_J(1), ..., \psi_1(k-1), .., \psi_J(k-1), \psi_1(k), .., \psi_J(k) \\ &= \Psi^J(0), \Psi^J(1), .., \Psi^J(k) \end{aligned}$$

ie., D^k represents all the (radar coordinate) track data available up to and including update k . Hence

$$P\left\{\lambda_{n_1..n_J}^{m_1..m_J} \middle| D^k\right\} \triangleq P\left\{\lambda_{n_1..n_J}^{m_1..m_J} \left| \Psi^J(0), \Psi^J(1), .., \Psi^J(k)\right.\right\}$$

Consider the updating of the probability of hypothesis $\lambda_{n_1..n_J}^{m_1..m_J}$ when the first track estimate for update k , (ie, $\psi_1(k)$) is considered. Using Bayes rule we have

$$\begin{aligned} P\left\{\lambda_{n_1..n_J}^{m_1..m_J} \middle| \psi_1(k), D^{k-1}\right\} &= \frac{p\left(\psi_1(k) \left| \lambda_{n_1..n_J}^{m_1..m_J}, D^{k-1}\right.\right) P\left\{\lambda_{n_1..n_J}^{m_1..m_J} \left| D^{k-1}\right.\right\}}{p\left(\psi_1(k) \left| D^{k-1}\right.\right)} \\ &= \frac{p\left(\psi_1(k) \left| \lambda_{n_1..n_J}^{m_1..m_J}, D^{k-1}\right.\right) P\left\{\lambda_{n_1..n_J}^{m_1..m_J} \left| D^{k-1}\right.\right\}}{\sum\limits_{\bar{n}_J=1}^{B_{n_1..n_{J-1}}} .. \sum\limits_{\bar{n}_1=1}^{1} \sum\limits_{\bar{m}_J=1}^{M_J} .. \sum\limits_{\bar{m}_1=1}^{M_1} p\left(\psi_1(k) \left| \lambda_{\bar{n}_1..\bar{n}_J}^{\bar{m}_1..\bar{m}_J}, D^{k-1}\right.\right) P\left\{\lambda_{\bar{n}_1..\bar{n}_J}^{\bar{m}_1..\bar{m}_J} \left| D^{k-1}\right.\right\}} \end{aligned}$$

Now, as noted earlier, in the implementation of MPTF the ground co-ordinate estimates $y_j^{m_j}(k)$ are used to perform the probability calculations, hence we need an equation in terms of them. Hence again, wherever we encounter the conditioning event $\theta_j^{m_j}$ we replace $\psi_j(k)$ with $y_j^{m_j}(k)$. Now, when we have the event $\lambda_{n_1..n_j..n_J}^{m_1..m_j..m_J}$ we note that this includes the event $\theta_j^{m_j}$ (as well as a number of other conditions). Hence the above equation can be replaced by

$$\begin{aligned} &P\left\{\lambda_{n_1..n_J}^{m_1..m_J} \middle| \psi_1(k), D^{k-1}\right\} = \\ &\frac{p\left(y_1^{m_1}(k) \left| \lambda_{n_1..n_J}^{m_1..m_J}, D^{k-1}\right.\right) P\left\{\lambda_{n_1..n_J}^{m_1..m_J} \left| D^{k-1}\right.\right\}}{\sum\limits_{\bar{n}_J=1}^{B_{n_1..n_{J-1}}} .. \sum\limits_{\bar{n}_1=1}^{1} \sum\limits_{\bar{m}_J=1}^{M_J} .. \sum\limits_{\bar{m}_1=1}^{M_1} p\left(y_1^{\bar{m}_1}(k) \left| \lambda_{\bar{n}_1..\bar{n}_J}^{\bar{m}_1..\bar{m}_J}, D^{k-1}\right.\right) P\left\{\lambda_{\bar{n}_1..\bar{n}_J}^{\bar{m}_1..\bar{m}_J} \left| D^{k-1}\right.\right\}} \end{aligned}$$

To calculate $p\left(y_1^{m_1}(k) \left| \lambda_{n_1..n_J}^{m_1..m_J}, D^{k-1}\right.\right)$, we need information regarding the dynamics of the target that $y_1^{m_1}(k)$ is associated with, or predictions to be provided by the individual trackers producing the tracks τ_j . The approach that will be taken will be the former, ie, using a model of target dynamics. The reasons for choosing this approach are

- Less information needs to be provided to the fusion algorithm by the trackers, reducing communication bandwidth.
- Using target dynamics gives a more general solution, for example if measurements are fused instead of track estimates as will be described later in the document, the approach can still be applied.

Now let us assume that target t_i satisfies the following state equation:

$$x_i(k) = F(k-1)\,x_i(k-1) + v(k-1) \qquad k = 1, 2, 3, \ldots \tag{13}$$

where $F(k-1)$ is the *state transition matrix* of each of the targets for the time interval between update $k-1$ and update k, and $v(k-1)$, $k = 1, 2, \ldots$ is the sequence of zero-mean white Gaussian process noise with covariance $E\left[v(k-1)\,v(k-1)'\right] = Q_{k-1}$. Hence, given the fused estimate $\bar{x}_i^H(k-1)$ for the hypothesis being considered, where H is the number of track estimates that were combined at time $k-1$ to obtain $\bar{x}_i^H(k-1)$, we have the following prediction at time k based on target dynamics

$$\begin{aligned} \bar{x}_i^0(k) &= F(k-1)\,\bar{x}_i^H(k-1) \\ \bar{P}_i^0(k) &= F(k-1)\,\bar{P}_i^H(k-1)\,F(k-1)' + Q_{k-1} \end{aligned} \tag{14}$$

Using the above we can easily obtain the likelihood

$$\begin{aligned} \Lambda_1^{m_1}(k) &\triangleq p\left(y_1^{m_1}(k)\,|\lambda_{n_1..n_J}^{m_1..m_J}, D^{k-1}\right) \\ &= \mathcal{N}\left(y_1^{m_1}(k); \bar{x}_i^0(k), T_1^{m_1}(k)\right) \\ &= \left|2\pi T_1^{m_1}(k)\right|^{-\frac{1}{2}} \exp\left\{-\frac{1}{2}\left(y_1^{m_1}(k) - \bar{x}_i^0(k)\right)' T_1^{m_1}(k)^{-1}\left(y_1^{m_1}(k) - \bar{x}_i^0(k)\right)\right\} \end{aligned}$$

where

$$T_1^{m_1}(k) = \bar{P}_i^0(k) - P_{i1}^{0m_1}(k) - P_{1i}^{m_1 0}(k) + P_1^{m_1}(k)$$

and

$$\begin{aligned} \bar{P}_i^0(k) &\triangleq E\left\{\tilde{x}_i^0(k)\,\tilde{x}_i^0(k)'\right\} \\ P_1^{m_1}(k) &\triangleq E\left\{\tilde{y}_1^{m_1}(k)\,\tilde{y}_1^{m_1}(k)'\right\} \\ P_{i1}^{0m_1}(k) &\triangleq E\left\{\tilde{x}_i^0(k)\,\tilde{y}_1^{m_1}(k)'\right\} \\ P_{1i}^{m_1 0}(k) &\triangleq E\left\{\tilde{y}_1^{m_1}(k)\,\tilde{x}_i^0(k)'\right\} \end{aligned}$$

In the above

$$\begin{aligned} \tilde{x}_i^0(k) &= x_i(k) - \bar{x}_i^0(k) \\ \tilde{y}_1^{m_1}(k) &= x_i(k) - y_1^{m_1}(k) \end{aligned}$$

Consider now, the updating of the probability of the hypothesis $\lambda_{n_1..n_j..n_J}^{m_1..m_j..m_J}$ when the j^{th} track estimate for update k, (ie, $\psi_j(k)$, $1 < j \leq J$) is considered. Using Bayes rule

we have

$$
\begin{aligned}
P\left\{\lambda_{n_1..n_J}^{m_1..m_J}\middle|\Psi^j(k),D^{k-1}\right\}
&= \frac{p\left(\psi_j(k)\,|\lambda_{n_1..n_J}^{m_1..m_J},\Psi^{j-1}(k),D^{k-1}\right)P\left\{\lambda_{n_1..n_J}^{m_1..m_J}\,|\Psi^{j-1}(k),D^{k-1}\right\}}{p\left(\psi_j(k)\,|\Psi^{j-1}(k),D^{k-1}\right)} \\
&= \frac{p\left(\psi_j(k)\,|\lambda_{n_1..n_J}^{m_1..m_J},\Psi^{j-1}(k),D^{k-1}\right)P\left\{\lambda_{n_1..n_J}^{m_1..m_J}\,|\Psi^{j-1}(k),D^{k-1}\right\}}{\sum_{\bar{n}_J=1}^{B_{n_1..n_{J-1}}}..\sum_{\bar{n}_1=1}^{1}\sum_{\bar{m}_J=1}^{M_J}..\sum_{\bar{m}_1=1}^{M_1}\left[\begin{array}{c} p\left(\psi_j(k)\,|\lambda_{\bar{n}_1..\bar{n}_J}^{\bar{m}_1..\bar{m}_J},\Psi^{j-1}(k),D^{k-1}\right)\times \\ P\left\{\lambda_{\bar{n}_1..\bar{n}_J}^{\bar{m}_1..\bar{m}_J}\,|\Psi^{j-1}(k),D^{k-1}\right\}\end{array}\right]}
\end{aligned}
$$

Now, as earlier, wherever we encounter the conditioning event $\theta_j^{m_j}$ we can replace $\psi_j(k)$ with $y_j^{m_j}(k)$. Again, when we have the event $\lambda_{n_1..n_j..n_J}^{m_1..m_j..m_J}$ we note that this includes the event $\theta_j^{m_j}$, hence the above equation can be replaced by

$$
P\left\{\lambda_{n_1..n_J}^{m_1..m_J}\middle|\Psi^j(k),D^{k-1}\right\} = \frac{p\left(y_j^{m_j}(k)\,|\lambda_{n_1..n_J}^{m_1..m_J},\Psi^{j-1}(k),D^{k-1}\right)P\left\{\lambda_{n_1..n_J}^{m_1..m_J}\,|\Psi^{j-1}(k),D^{k-1}\right\}}{\sum_{\bar{n}_J=1}^{B_{n_1..n_{J-1}}}..\sum_{\bar{n}_1=1}^{1}\sum_{\bar{m}_J=1}^{M_J}..\sum_{\bar{m}_1=1}^{M_1}\left[\begin{array}{c} p\left(y_j^{\bar{m}_j}(k)\,|\lambda_{\bar{n}_1..\bar{n}_J}^{\bar{m}_1..\bar{m}_J},\Psi^{j-1}(k),D^{k-1}\right)\times \\ P\left\{\lambda_{\bar{n}_1..\bar{n}_J}^{\bar{m}_1..\bar{m}_J}\,|\Psi^{j-1}(k),D^{k-1}\right\}\end{array}\right]}
$$

Then we can easily obtain the likelihood

$$
\begin{aligned}
\Lambda_j^{m_j}(k) &\triangleq p\left(y_j^{m_j}(k)\,|\lambda_{n_1..n_J}^{m_1..m_J},\Psi^{j-1}(k),D^{k-1}\right) \\
&= \mathcal{N}\left(y_j^{m_j}(k);\bar{x}_i^h(k),T_j^{m_j}(k)\right) \\
&= \left|2\pi T_j^{m_j}(k)\right|^{-\frac{1}{2}}\exp\left\{-\frac{1}{2}\left(y_j^{m_j}(k)-\bar{x}_i^h(k)\right)'T_j^{m_j}(k)^{-1}\left(y_j^{m_j}(k)-\bar{x}_i^h(k)\right)\right\}
\end{aligned}
$$

where

$$
T_j^{m_j}(k)=\bar{P}_i^h(k)-P_{ij}^{hm_j}(k)-P_{ji}^{m_jh}(k)+P_j^{m_j}(k)
$$

and

$$
\begin{aligned}
\bar{P}_i^h(k) &\triangleq E\left\{\tilde{x}_i^h(k)\,\tilde{x}_i^h(k)'\right\} \\
P_j^{m_j}(k) &\triangleq E\left\{\tilde{y}_j^{m_j}(k)\,\tilde{y}_j^{m_j}(k)'\right\} \\
P_{ij}^{hm_j}(k) &\triangleq E\left\{\tilde{x}_i^h(k)\,\tilde{y}_j^{m_j}(k)'\right\} \\
P_{ji}^{m_jh}(k) &\triangleq E\left\{\tilde{y}_j^{m_j}(k)\,\tilde{x}_i^h(k)'\right\}
\end{aligned}
$$

In the above

$$
\begin{aligned}
\tilde{x}_i^h(k) &= x_i(k)-\bar{x}_i^h(k) \\
\tilde{y}_j^{m_j}(k) &= x_i(k)-y_j^{m_j}(k)
\end{aligned}
$$

and $\bar{x}_i^h(k)$ is the most recent fused estimate/prediction for target t_i for the hypothesis being considered.

Hence combining the cases of $j = 1$, $1 < j \leq J$ and again assuming that no two resolved ground tracks which are due to the same target can be associated with the same propagation path gives the following probability update for the hypothesis $\lambda_{n_1..n_j..n_J}^{m_1..m_j..m_J}$ using the j^{th} track estimate of update k, $\psi_j(k)$:

$$P\left\{\lambda_{n_1..n_J}^{m_1..m_J}\left|\Psi^j(k), D^{k-1}\right.\right\} = \frac{\Lambda_j^{m_j}(k)\, P\left\{\lambda_{n_1..n_J}^{m_1..m_J}\left|\Psi^{j-1}(k), D^{k-1}\right.\right\}}{\sum\limits_{\bar{n}_1=1}^{1} .. \sum\limits_{\bar{n}_J=1}^{B_{n_1..n_{J-1}}} \sum\limits_{\bar{m}_1=1}^{M_1} .. \sum\limits_{\substack{\bar{m}_J=1 \\ \bar{m}_J \notin S_J}}^{M_J} \Lambda_j^{\bar{m}_j}(k)\, P\left\{\lambda_{\bar{n}_1..\bar{n}_J}^{\bar{m}_1..\bar{m}_J}\left|\Psi^{j-1}(k), D^{k-1}\right.\right\}} \tag{15}$$

where, for $j = 1$:

$$\begin{aligned}
\Lambda_j^{m_j}(k) &= \Lambda_1^{m_1}(k) \triangleq p\left(y_1^{m_1}(k)\,|\lambda_{n_1..n_J}^{m_1..m_J}, D^{k-1}\right) \\
\Lambda_j^{\bar{m}_j}(k) &= \Lambda_1^{\bar{m}_1}(k) \triangleq p\left(y_1^{\bar{m}_1}(k)\,|\lambda_{\bar{n}_1..\bar{n}_J}^{\bar{m}_1..\bar{m}_J}, D^{k-1}\right)
\end{aligned}$$

and for $1 < j \leq J$:

$$\begin{aligned}
\Lambda_j^{m_j}(k) &\triangleq p\left(y_j^{m_j}(k)\,|\lambda_{n_1..n_J}^{m_1..m_J}, \Psi^{j-1}(k), D^{k-1}\right) \\
\Lambda_j^{\bar{m}_j}(k) &\triangleq p\left(y_j^{\bar{m}_j}(k)\,|\lambda_{\bar{n}_1..\bar{n}_J}^{\bar{m}_1..\bar{m}_J}, \Psi^{j-1}(k), D^{k-1}\right)
\end{aligned}$$

The likelihood for both cases is then

$$\Lambda_j^{m_j}(k) = \left|2\pi T_j^{m_j}(k)\right|^{-\frac{1}{2}} \exp\left[-\frac{1}{2}\nu_j^{m_j}(k)' T_j^{m_j}(k)^{-1} \nu_j^{m_j}(k)\right] \tag{16}$$

where

$$\begin{aligned}
\nu_j^{m_j}(k) &\triangleq \left(y_j^{m_j}(k) - \bar{x}_i^h(k)\right) \\
T_j^{m_j}(k) &= \bar{P}_i^h(k) - P_{ij}^{hm_j}(k) - P_{ji}^{m_jh}(k) + P_j^{m_j}(k)
\end{aligned} \tag{17}$$

and

$$\begin{aligned}
\bar{P}_i^h(k) &= E\left\{\tilde{x}_i^h(k)\,\tilde{x}_i^h(k)'\right\} \\
P_j^{m_j}(k) &= E\left\{\tilde{y}_j^{m_j}(k)\,\tilde{y}_j^{m_j}(k)'\right\} \\
P_{ij}^{hm_j}(k) &= E\left\{\tilde{x}_i^h(k)\,\tilde{y}_j^{m_j}(k)'\right\} \\
P_{ji}^{m_jh}(k) &= E\left\{\tilde{y}_j^{m_j}(k)\,\tilde{x}_i^h(k)'\right\} \\
\tilde{x}_i^h(k) &= x_i(k) - \bar{x}_i^h(k) \\
\tilde{y}_j^{m_j}(k) &= x_i(k) - y_j^{m_j}(k)
\end{aligned}$$

In the above, $\bar{x}_i^h(k)$ is the most recent fused estimate/prediction for the target t_i that $y_j^{m_j}(k)$ is assumed to be associated with for the hypothesis being considered. Note that $\bar{x}_i^0(k)$, $k > 0$ are predictions based on fused data available up to time $k-1$, whereas

$\bar{x}_i^h(k)$, $h \neq 0$, are fused estimates. The predictions $\bar{x}_i^0(k)$ are used when no fused estimates are available for target t_i at time k.

Consider now the estimate $\bar{x}_i^{h+1}(k)$ of $x_i(k)$ and its covariance $\bar{P}_i^{h+1}(k)$ obtained by the fusion of $\bar{x}_i^h(k), h = 0, 1, .., H-1$ with $y_j^{m_j}(k)$. This is again derived using the fundamental equations of linear estimation and essentially the same reasoning as shown earlier for $k = 0$.

The resulting fusion equations are hence

$$\bar{x}_i^{h+1}(k) = \bar{x}_i^h(k) + \left(\bar{P}_i^h(k) - P_{ij}^{hm_j}(k)\right) T_j^{m_j}(k)^{-1} \left(y_j^{m_j}(k) - \bar{x}_i^h(k)\right) \tag{18}$$

$$\bar{P}_i^{h+1}(k) = \bar{P}_i^h(k) - \left(\bar{P}_i^h(k) - P_{ij}^{hm_j}(k)\right) T_j^{m_j}(k)^{-1} \left(\bar{P}_i^h(k) - P_{ji}^{m_jh}(k)\right) \tag{19}$$

Some comments are warranted regarding a further point of divergence from the work described in [17], [18] and [19]. In those papers, a description is given of the application of Gaussian mixture equations [4] for describing the probability density function of fused tracks. The idea is, given a set of path dependent hypotheses which only differ in their path assignments (ie, they all correspond to the same path independent hypothesis), one can construct a fused estimate for the corresponding path independent hypothesis which is a Gaussian mixture of the estimates for the path dependent hypotheses. This idea developed largely from a limitation of the static algorithm. Because the hypothesis probabilities in the static algorithm are based on a single update of the tracks, there is usually not enough information available to achieve a single dominant path dependent hypothesis. Usually, however, if one groups the path dependent hypotheses into their corresponding path independent hypotheses, the result is a smaller number of hypotheses with significant probabilities. This results in greater ease of presentation and interpretation of data. This approach does have a down side, however. Knowing the correct path to be associated with a particular track in radar coordinates is critical to accurately mapping it to ground coordinates, and hence obtaining the best fused estimate. The Gaussian mixture approach as applied to this problem does not achieve this. Instead it simply gives a "best" estimate in a statistical sense based on the relative probabilities of the propagation paths. In a sense, this estimate is always wrong, in that it does not correspond to any particular path transformation. Furthermore the relative probabilities of grouped path dependent hypotheses in the static algorithm generally vary significantly from one update to the next. This leads to undesirable "jitter" in the fused tracks corresponding to the path independent hypotheses. With the introduction of the dynamic algorithm this conundrum is largely resolved. Because the DMPTF algorithm determines its hypothesis probabilities over time, much more data is available for determining relative hypothesis probabilities. The outcome is that after the initiation of a hypothesis tree, a small number (often one) of path dependent hypotheses quickly become dominant. The result is that there is no need for calculating Gaussian mixtures of the path dependent hypothesis estimates to determine equivalent path independent estimates. As a result the aforementioned approach was discarded.

6.2 Dealing with Changes in the Number of Tracks

A common phenomenon that occurs is the dropping out and initiation of multipath tracks as ionospheric conditions change with time. Without being able to deal with this phenomenon, the DMPTF algorithm would have little application. This section deals with additions to the DMPTF algorithm which enable it to cope with both increases and decreases in number of ground tracks with time.

First let us consider what happens when the number of tracks increases from update k to $k+1$. The approach to take in this situation is to predict the hypothesis estimates from update k to update $k+1$ exactly as for the case when the number of tracks does not change, using equations 15, 16, 18, 19, and then to extend the tree using the newly started tracks $\tau_{J(k)+j}$, $j = 1, .., [J(k+1) - J(k)]$. The tree is extended in the same way as was done during the initial creation of the original tree, using equations 2, 3, 5, 7, and 8.

Now consider what happens when the number of tracks decreases from update k to $k+1$. Let us start with a simple example to help us visualize what needs to be done when the number of tracks decreases. Consider the case of 3 tracks, 2 paths per track (only OTHR tracks) at time k . Now look at the sample space Ω, of all possible outcomes (where each path dependent hypothesis is a single outcome) as shown below. Note that path dependent hypotheses that have two tracks with the same propagation path for the same target are included (they are not in the actual implementation).

$$\Omega = \{\lambda_{111}, \lambda_{112}, \lambda_{121}, \lambda_{122}, \lambda_{123}\}$$

where

$$\begin{aligned}
\lambda_{111} &= \{\lambda_{111}^{111}, \lambda_{111}^{121}, \lambda_{111}^{211}, \lambda_{111}^{221}, \lambda_{111}^{112}, \lambda_{111}^{122}, \lambda_{111}^{212}, \lambda_{111}^{222}\} \\
\lambda_{112} &= \{\lambda_{112}^{111}, \lambda_{112}^{121}, \lambda_{112}^{211}, \lambda_{112}^{221}, \lambda_{112}^{112}, \lambda_{112}^{122}, \lambda_{112}^{212}, \lambda_{112}^{222}\} \\
\lambda_{121} &= \{\lambda_{121}^{111}, \lambda_{121}^{121}, \lambda_{121}^{211}, \lambda_{121}^{221}, \lambda_{121}^{112}, \lambda_{121}^{122}, \lambda_{121}^{212}, \lambda_{121}^{222}\} \\
\lambda_{122} &= \{\lambda_{122}^{111}, \lambda_{122}^{121}, \lambda_{122}^{211}, \lambda_{122}^{221}, \lambda_{122}^{112}, \lambda_{122}^{122}, \lambda_{122}^{212}, \lambda_{122}^{222}\} \\
\lambda_{123} &= \{\lambda_{123}^{111}, \lambda_{123}^{121}, \lambda_{123}^{211}, \lambda_{123}^{221}, \lambda_{123}^{112}, \lambda_{123}^{122}, \lambda_{123}^{212}, \lambda_{123}^{222}\}
\end{aligned}$$

Now, the above is only one possible grouping of path dependent hypotheses. Let us now group all the path dependent hypotheses that have identical first and last target numbers, ie, ignoring the target number of the second track. This is performed to see what needs to be done if, say, track 2 no longer exists at time $k+1$. Note that we will treat target number 3 as being the same as target number 2 for the purpose of grouping. The reason that this can be done is that during hypothesis tree generation the target numbers are incremented simply to signify that a different target is introduced. Once the target number of the second track becomes irrelevant, as is the case here, the third track can only be due to the same target or a different target to the first, and thus can simply be given a label of 1 or 2, the former label applying when it is the same target and the latter when it is not. We then obtain the following

$$\Omega = \left\{\lambda_{1*1}, \lambda_{1*(2,3)}\right\}$$

The two outcomes listed on the right hand side of the above equations can be decomposed into the following

$$\lambda_{1*1} = \left\{\lambda_{1*1}^{1*1}, \lambda_{1*1}^{2*1}, \lambda_{1*1}^{1*2}, \lambda_{1*1}^{2*2}\right\}$$

where

$$\begin{aligned}
\lambda_{1*1}^{1*1} &= \left\{\lambda_{111}^{111}, \lambda_{111}^{121}, \lambda_{121}^{111}, \lambda_{121}^{121}\right\} \\
\lambda_{1*1}^{2*1} &= \left\{\lambda_{111}^{211}, \lambda_{111}^{221}, \lambda_{121}^{211}, \lambda_{121}^{221}\right\} \\
\lambda_{1*1}^{1*2} &= \left\{\lambda_{111}^{112}, \lambda_{111}^{122}, \lambda_{121}^{112}, \lambda_{121}^{122}\right\} \\
\lambda_{1*1}^{2*2} &= \left\{\lambda_{111}^{212}, \lambda_{111}^{222}, \lambda_{121}^{212}, \lambda_{121}^{222}\right\}
\end{aligned}$$

and

$$\lambda_{1*(2,3)} = \left\{\lambda_{1*(2,3)}^{1*1}, \lambda_{1*(2,3)}^{2*1}, \lambda_{1*(2,3)}^{1*2}, \lambda_{1*(2,3)}^{2*2}\right\}$$

where

$$\begin{aligned}
\lambda_{1*(2,3)}^{1*1} &= \left\{\lambda_{112}^{111}, \lambda_{112}^{121}, \lambda_{122}^{111}, \lambda_{122}^{121}, \lambda_{123}^{111}, \lambda_{123}^{121}\right\} \\
\lambda_{1*(2,3)}^{2*1} &= \left\{\lambda_{112}^{211}, \lambda_{112}^{221}, \lambda_{122}^{211}, \lambda_{122}^{221}, \lambda_{123}^{211}, \lambda_{123}^{221}\right\} \\
\lambda_{1*(2,3)}^{1*2} &= \left\{\lambda_{112}^{112}, \lambda_{112}^{122}, \lambda_{122}^{112}, \lambda_{122}^{122}, \lambda_{123}^{112}, \lambda_{123}^{122}\right\} \\
\lambda_{1*(2,3)}^{2*2} &= \left\{\lambda_{112}^{212}, \lambda_{112}^{222}, \lambda_{122}^{212}, \lambda_{122}^{222}, \lambda_{123}^{212}, \lambda_{123}^{222}\right\}
\end{aligned}$$

From the above, it is clear that

$$P\left\{\lambda_{1*1}\right\} = P\left\{\lambda_{1*1}^{1*1}\right\} + P\left\{\lambda_{1*1}^{2*1}\right\} + P\left\{\lambda_{1*1}^{1*2}\right\} + P\left\{\lambda_{1*1}^{2*2}\right\}$$

where

$$\begin{aligned}
P\left\{\lambda_{1*1}^{1*1}\right\} &= P\left\{\lambda_{111}^{111}\right\} + P\left\{\lambda_{111}^{121}\right\} + P\left\{\lambda_{121}^{111}\right\} + P\left\{\lambda_{121}^{121}\right\} \\
P\left\{\lambda_{1*1}^{2*1}\right\} &= P\left\{\lambda_{111}^{211}\right\} + P\left\{\lambda_{111}^{221}\right\} + P\left\{\lambda_{121}^{211}\right\} + P\left\{\lambda_{121}^{221}\right\} \\
P\left\{\lambda_{1*1}^{1*2}\right\} &= P\left\{\lambda_{111}^{112}\right\} + P\left\{\lambda_{111}^{122}\right\} + P\left\{\lambda_{121}^{112}\right\} + P\left\{\lambda_{121}^{122}\right\} \\
P\left\{\lambda_{1*1}^{2*2}\right\} &= P\left\{\lambda_{111}^{212}\right\} + P\left\{\lambda_{111}^{222}\right\} + P\left\{\lambda_{121}^{212}\right\} + P\left\{\lambda_{121}^{222}\right\}
\end{aligned}$$

and

$$P\left\{\lambda_{1*(2,3)}\right\} = P\left\{\lambda_{1*(2,3)}^{1*1}\right\} + P\left\{\lambda_{1*(2,3)}^{2*1}\right\} + P\left\{\lambda_{1*(2,3)}^{1*2}\right\} + P\left\{\lambda_{1*(2,3)}^{2*2}\right\}$$

where

$$\begin{aligned}
P\left\{\lambda_{1*(2,3)}^{1*1}\right\} &= P\left\{\lambda_{112}^{111}\right\} + P\left\{\lambda_{112}^{121}\right\} + P\left\{\lambda_{122}^{111}\right\} + P\left\{\lambda_{122}^{121}\right\} + P\left\{\lambda_{123}^{111}\right\} + P\left\{\lambda_{123}^{121}\right\} \\
P\left\{\lambda_{1*(2,3)}^{2*1}\right\} &= P\left\{\lambda_{112}^{211}\right\} + P\left\{\lambda_{112}^{221}\right\} + P\left\{\lambda_{122}^{211}\right\} + P\left\{\lambda_{122}^{221}\right\} + P\left\{\lambda_{123}^{211}\right\} + P\left\{\lambda_{123}^{221}\right\} \\
P\left\{\lambda_{1*(2,3)}^{1*2}\right\} &= P\left\{\lambda_{112}^{112}\right\} + P\left\{\lambda_{112}^{122}\right\} + P\left\{\lambda_{122}^{112}\right\} + P\left\{\lambda_{122}^{122}\right\} + P\left\{\lambda_{123}^{112}\right\} + P\left\{\lambda_{123}^{122}\right\} \\
P\left\{\lambda_{1*(2,3)}^{2*2}\right\} &= P\left\{\lambda_{112}^{212}\right\} + P\left\{\lambda_{112}^{222}\right\} + P\left\{\lambda_{122}^{212}\right\} + P\left\{\lambda_{122}^{222}\right\} + P\left\{\lambda_{123}^{212}\right\} + P\left\{\lambda_{123}^{222}\right\}
\end{aligned}$$

If we now rename the hypotheses as follows

$$\lambda_{n_1 * n_3}^{m_1 * m_3} \rightarrow \lambda_{n_1 \min(n_3)}^{m_1 m_3}$$

we have the leaves of the hypothesis tree for two tracks and their corresponding probabilities. This regrouping can be done at update k once it is known that track 2 will terminate at update $k+1$.

Now consider the target state estimates by looking at some examples. Consider the new hypothesis $\lambda_{11}^{11} \triangleq \lambda_{1*1}^{1*1}$

$$\lambda_{11}^{11} \triangleq \lambda_{1*1}^{1*1} = \{\lambda_{111}^{111}, \lambda_{111}^{121}, \lambda_{121}^{111}, \lambda_{121}^{121}\}$$

In the above grouped hypothesis, λ_{111}^{111} and λ_{111}^{121} have one target estimate $\bar{x}_1(k)$ associated with them, whereas λ_{121}^{111} and λ_{121}^{121} have 2 associated target estimates $\bar{x}_1(k)$, $\bar{x}_2(k)$. The new grouped hypothesis should only have target estimates that involve tracks 1 and 3, since target estimates that *only* involve track 2 must be terminated when track 2 terminates. Now, in all of $\lambda_{111}^{111}, \lambda_{111}^{121}, \lambda_{121}^{111}, \lambda_{121}^{121}$, estimate $\bar{x}_1(k)$ involves tracks 1 and 3 (and also track 2 for $\lambda_{111}^{111}, \lambda_{111}^{121}$) hence $\bar{x}_1(k)$ is retained. Estimate $\bar{x}_2(k)$ in $\lambda_{121}^{111}, \lambda_{121}^{121}$ only involves track 2 and hence is removed. Now we are left with four estimates $\bar{x}_1(k)$, one for each of the path dependent hypotheses. They are highly correlated, with the extent of the correlation again being extremely difficult to ascertain. It is proposed that the best approach here is to compute the equivalent Gaussian to this Gaussian mixture to form the new $\bar{x}_1(k)$. Their high correlation is expected to generally result in their means being fairly close in value and hence the approximation in forming the equivalent Gaussian should be reasonably good.

Now consider the new hypothesis $\lambda_{12}^{11} \triangleq \lambda_{1*(2,3)}^{1*1}$

$$\lambda_{12}^{11} \triangleq \lambda_{1*(2,3)}^{1*1} = \{\lambda_{112}^{111}, \lambda_{112}^{121}, \lambda_{122}^{111}, \lambda_{122}^{121}, \lambda_{123}^{111}, \lambda_{123}^{121}\}$$

In the above grouped hypothesis

- There are no hypotheses that have only one target estimate associated with them.
- Hypotheses λ_{112}^{111}, λ_{112}^{121} λ_{122}^{111}, λ_{122}^{121} have two target estimates $\bar{x}_1(k)$, $\bar{x}_2(k)$ associated with them. Neither of the estimates *only* involve track 2 in any of the hypotheses, hence both estimates are retained.
- Hypotheses λ_{123}^{111} and λ_{123}^{121} have 3 associated target estimates $\bar{x}_1(k)$, $\bar{x}_2(k)$, $\bar{x}_3(k)$. Estimate $\bar{x}_2(k)$ *only* involves track 2 and hence must be removed.

Now we are left with six estimates $\bar{x}_1(k)$; one for each of the path dependent hypotheses. We then compute the equivalent Gaussian to this Gaussian mixture to form the new $\bar{x}_1(k)$. We are also left with four estimates $\bar{x}_2(k)$, one for each of λ_{112}^{111} , λ_{112}^{121} , λ_{122}^{111} , λ_{122}^{121} , and two estimates $\bar{x}_3(k)$, one for each of λ_{123}^{111} and λ_{123}^{121} . Now estimates $\bar{x}_3(k)$ for each of λ_{123}^{111} and λ_{123}^{121} are associated with track 3, and estimates $\bar{x}_2(k)$ for each of λ_{112}^{111} , λ_{112}^{121} , λ_{122}^{111} , λ_{122}^{121} are also (at least in part) associated with track 3. We hence compute the equivalent Gaussian to this Gaussian mixture (of 6 Gaussians) to form the new $\bar{x}_2(k)$.

We can now generalize from the above specific case to propose a simple general algorithm for performing the regrouping for any number of tracks and propagation paths as follows. Starting with a set of path dependent hypotheses $\lambda_{n_1..n_K..n_J}^{m_1..m_K..m_J}(k)$ where track τ_K terminates, perform the following steps:

1. Rename each of $\lambda_{n_1..n_K..n_J}^{m_1..m_K..m_J}(k)$ by removing subscript n_K and superscript m_K. Delete all target estimates that *only* involve a contribution from τ_K. Rename tracks $\tau_{K+1}, .., \tau_J$ to $\tau_K, .., \tau_{J-1}$.
2. For each of the newly named $\lambda_{n_1..n_{J-1}}^{m_1..m_{J-1}}(k)$ (where subscript n_K and superscript m_K are now removed, and tracks $\tau_{K+1}, .., \tau_J$ are renamed $\tau_K, .., \tau_{J-1}$), rename the hypothesis using the most compact notation that allows description of tracks introducing new targets or tracks being associated with previously hypothesised targets. Reordering of subscript numbers must be performed for some hypotheses so that the first new target identifier is 1, the second is 2, etc. Rename the target estimates associated with each hypothesis as the corresponding hypothesis is renamed.
3. Group all hypotheses that now have identical labels (subscripts and superscripts) to form new merged hypotheses. The probabilities of the new hypotheses are set to the sum of the probabilities of the hypotheses from which they are formed.
4. For each new merged hypothesis, form the equivalent Gaussian from the (unnormalised) Gaussian mixture that corresponds to the hypotheses from which the merged hypothesis is formed.

If the number of tracks decreases by more than one, the above grouping is repeated for each track that has terminated. The hypothesis tree is then ready for the prediction step from update k to $k+1$ which is performed as described previously.

The equations for the replacement of the unnormalised Gaussian mixture associated with a group of hypotheses by a single Gaussian probability density function are given by the following. Consider N estimates, $\bar{x}_j$, with covariances $\bar{P}_j$ and probabilities p_j, $j = 1, .., N$, where the hypotheses that the estimates are associated with are mutually exclusive but *not* exhaustive, that is $p = \sum_{j=1}^{N} p_j \neq 1$. The mean that is used for approximating the Gaussian distribution is:

$$\bar{x} = \frac{1}{p} \sum_{j=1}^{N} p_j \bar{x}_j \tag{20}$$

and its associated covariance is:

$$P = \frac{1}{p} \left(\sum_{j=1}^{N} p_j \bar{P}_j + \tilde{P} \right) \tag{21}$$

where

$$\tilde{P} \triangleq \sum_{j=1}^{N} (\bar{x}_j - \bar{x}) (\bar{x}_j - \bar{x})' p_j$$

Note that the factor $1/p$ which appears in front of the sum for both the estimate and covariance fusion normalises the hypothesis probabilities with respect to the probability of the merged hypotheses. Equations 20 and 21 are referenced from [4] page 47, where there is some explanation of their derivation as well. Note that they have been modified to account for the fact that the hypotheses are not exhaustive.

To demonstrate how the above algorithm works in a slightly more complicated case, consider the application of the algorithm to two specific example hypotheses from a hypothesis tree for six radar-coordinate tracks. Consider the hypotheses $\lambda_{123123}^{123456}$ and $\lambda_{123143}^{123456}$ with probabilities p_a, and p_b respectively. The former has three target estimates, $\bar{x}_1, \bar{x}_2, \bar{x}_3$ associated with it, whereas the latter hypothesis has four, ie $\bar{x}_1, \bar{x}_2, \bar{x}_3, \bar{x}_4$. Let track 2 terminate. After step 1 the new hypothesis names are λ_{13123}^{13456} and λ_{13143}^{13456} respectively. For the former all three target estimates $\bar{x}_1, \bar{x}_2, \bar{x}_3$ remain, whereas for the later the estimate $\bar{x}_2$ is removed leaving $\bar{x}_1, \bar{x}_3, \bar{x}_4$. After step 2 the two hypotheses have identical hypothesis names and estimate labels, ie λ_{12132}^{13456} and $\bar{x}_1, \bar{x}_2, \bar{x}_3$ respectively. Hence the two hypotheses are then merged to form the new hypothesis λ_{12132}^{13456} with probability $p = p_a + p_b$. The formation of the three Gaussian mixtures and their equivalent Gaussians is performed by a straight forward application of equations 20 and 21.

6.3 Track Dependence

The likelihood and fused estimate equations in both the static and dynamic cases incorporate cross-covariance terms so that, in theory, dependence between estimates $\psi_j(k)$ can be accommodated. The main causes of dependence between tracks are

- common process noise [5],
- tracks being updated using common measurements [19], [7], [13], and
- common propagation path segments [19].

In practice, however, it was found that the cross-covariance terms are extremely difficult to determine because of the complexity of the interactions involved. Research was undertaken to determine the cross-covariance terms, or at least to approximate them, with at this stage little success.

The approaches for dealing with dependence between tracks that were looked at were

1. Assume independence, ie, set cross-covariances to zero.
2. Directly calculate cross covariances.
3. Simple approximations to cross-covariances, based on judgement.
4. The covariance intersection algorithm [27].
5. Rearrangement of equations to reconstruct measurements.
6. Using the measurements from which the original unfused tracks have been formed.

A summary of the approaches and the results obtained upon testing is given in the following paragraphs.

The first approach, which is also the easiest, is to simply ignore the interdependencies between the multipath tracks, ie, to set the cross-covariance terms in the likelihood calculations and fusion equations to zero. This was tested and used as a reference against which the performance of the other approaches was compared. Somewhat surprisingly, while obviously not optimal, the approach was found to provide reasonable fusion and likelihood calculations, with the major negative effect being that the covariances of the fused estimates shrink somewhat too quickly over time. This was found to result in a particular favoured hypothesis having its probability approach 1 more rapidly than would be expected. It also led to the resulting estimates being very "smooth", when compared with the original tracks, again more so than an optimal algorithm would be expected to produce.

The second approach as suggested in [19], and described in [5] is extremely unwieldy when applied to DMPTF, because of the number of tracks that need to be fused and the asynchronicity that must be accommodated. This technique also puts restrictions on the models used by the trackers that feed information to the DMPTF algorithm. Additionally, the approach would, if in fact it could be implemented, at best, only solve the process noise aspect of the dependence. For these reasons, this technique was not considered further.

The third approach is to provide a simple approximation for the cross-covariance matrices, as suggested in [19], and [5]. Considerable experimentation was performed with this technique, but it was found that this approach simply did not work for DMPTF. A geometric combination [5] p455 of the covariances of the two dependent estimates as well as several other types of approximation of the cross-covariance terms were tried, none of which came even close to giving satisfactory performance. The resulting covariance sums (shown in equations 4 and 17) would sooner or later result in a non-singular covariance matrix, leading the DMPTF software execution to terminate. The only way that this could be avoided was by setting the cross-covariance terms to values very close to zero, effectively approximating the first approach listed above. The suspected reason is that the dependence between new data and previously fused data varied substantially with time and between hypotheses, making it impossible for a simple cross-covariance approximation to be used successfully.

The fourth option considered is the covariance intersection (CI) algorithm [27] (also referred to as Gaussian Intersection). The aim of this approach is to avoid the need to calculate or estimate dependence between data which is to be fused, while still arriving at a result that is optimal in some sense. A short summary of the algorithm and some experimental findings will be given here; full details of the technique can be found in [27].

Consider two estimates, x_a and x_b with means a and b and covariances P_{aa} and P_{bb} respectively. The covariance intersection algorithm forms the fused estimate c with covariance P_{cc} by using the following equations

$$\begin{aligned} P_{cc}^{-1} &= \omega P_{aa}^{-1} + (1-\omega) P_{bb}^{-1} \\ P_{cc}^{-1} c &= \omega P_{aa}^{-1} a + (1-\omega) P_{bb}^{-1} b \qquad \text{where } \omega \in [0,1] \end{aligned} \tag{22}$$

The free parameter ω can be used to optimize the fused estimate with respect to different performance criteria such as minimizing the trace or the determinant of P_{cc}. The result,

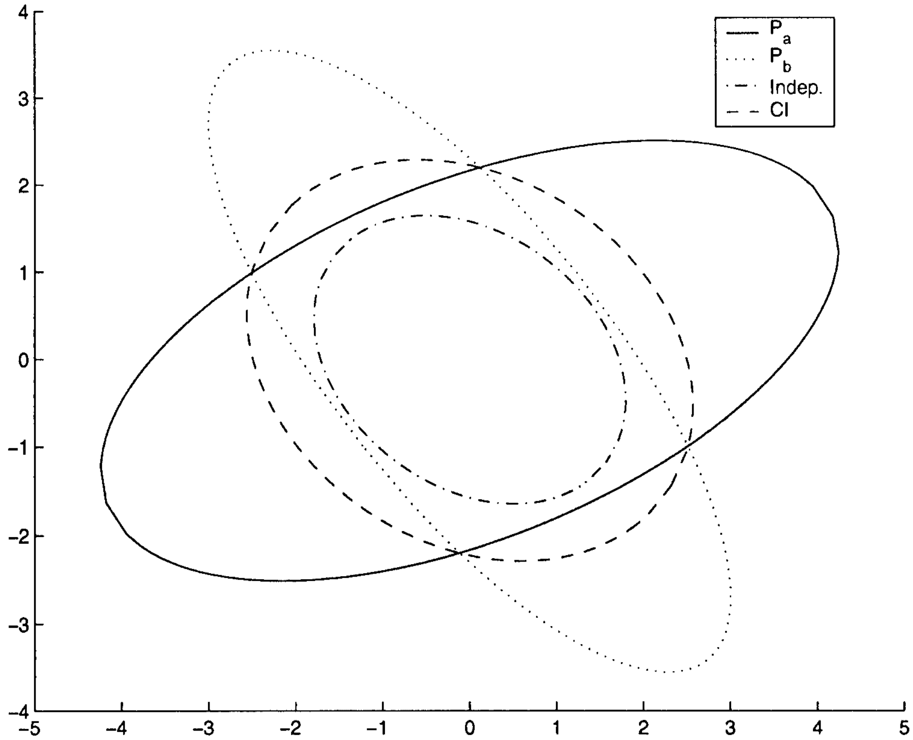

Figure 6: Example of fusion using the Covariance Intersection Technique

it is argued, is a conservative and non-divergent update of the fused estimate. Figure 6 shows a simple example of the fused estimate produced by the covariance intersection technique. Three-sigma ellipses for two coincident estimates are shown (labelled P_a and P_b), as well as two ellipses representing the fused estimate. The Ellipse labelled "Indep." represents the fused estimate obtained by assuming that the estimates x_a and x_b are independent. The ellipse labelled "CI" represents the fused estimate obtained using the covariance intersection technique with determinant minimization.

The CI technique looked quite promising initially; however, in testing done to date its performance has been disappointing. Updating of estimates with ω set to minimize the trace as well as the determinant of the fused estimate's covariance (within the constraints of equations 22) was implemented in the DMPTF algorithm, and tested. The outcome was that when determinant minimisation was used, the CI algorithm would usually either select one or other of the two estimates (ie, $\omega \approx 0$, or $\omega \approx 1$) as its fused estimate, and tended to jump between the two in subsequent updates. Using trace minimization provided a more stable result, but once again the algorithm would choose one of the associated estimates as the fused estimate, rather than providing a "weighted average" of the two. While testing performed to date has shown poor results, leading to the technique being put aside, further investigation to analyse the reasons for the unsatisfactory performance may be worthwhile, and possibly lead to improvements in performance.

The fifth option considered was to perform measurement reconstruction [15], [16], by manipulation of the Kalman filter equations which are used to perform tracking. This approach offers the opportunity to account for the effects of common process noise without calculating the cross-covariances. It would not account for the effects of tracks being updated with common measurements or the common propagation path-segment effects. For our application, it was found to offer no advantages over simply using the measurements themselves, since using the measurements would be no more difficult than accessing the information required for measurement reconstruction. Measurement reconstruction also has the disadvantage of not being able to account for common measurements as can obviously be done if the actual measurements are used in the fusion process, and is more difficult to generalize to a multi-sensor application where each sensor could potentially use a different Kalman filter (or some other filter for that matter). As a result this approach was not investigated further.

The sixth approach that was considered was to associate and fuse *the measurements from which the radar coordinate tracks were formed.* It should be emphasised here that only the measurements which have already been selected by the tracking algorithm/s are considered, thus avoiding the computational load associated with the centralized tracking approach. Doing this immediately avoids the dependence due to common process noise, and offers the potential to remove the effects of common measurements by simply inspecting the measurements prior to fusion. This approach still does not address the common propagation path segment contribution to track dependence, notwithstanding that this contribution may be less difficult to deal with once the other effects are removed. The approach has considerable merit; hence the association and fusion equations required to use measurements have been derived and are presented later in this report. The approach has not as yet been implemented for testing however.

To date, the first option, ie, assume that the data is independent, is what is implemented in the DMPTF algorithm.

7 Performance Testing of DMPTF Algorithm

The DMPTF algorithm has been implemented in C++ and integrated into a development test-bed in order to determine its performance and assess further development requirements.

For the initial testing of the DMPTF algorithm, pruning was set to retain the 100 highest probability hypotheses, and the cross-covariance terms of equations 4 and 17 were set to zero. Stored examples of real data from the Jindalee OTHR at Alice Springs, Australia were used as inputs to the algorithm and performance ascertained. Two types of ionospheric coordinate registration (CR) data were used for the path transformations. Actual CR data based on ionospheric sounder measurements was used, as well as data based on a simplified two-layer ionospheric model. The reason for using the simplified model in some of the testing was to enable control over more parameters.

During the early testing it became evident that performance could be improved by making some minor modifications to the DMPTF algorithm. It should be noted that the changes to the algorithm were essentially of an empirical nature, giving improved performance for the application in question, but not adding to the theory. The first modification was made because in some cases, if a hypothesis which was originally of highest probability became inappropriate several updates later, the algorithm would sometimes not readjust sufficiently quickly. The cause of this was that the probabilities of non-favoured hypotheses would become extremely small after several updates, thus making it difficult for them to quickly become the highest probability hypothesis when conditions changed. The remedy for this was to set a minimum value of the probability for each hypothesis. At every update the hypothesis probabilities would be calculated using the equations outlined in the earlier sections, then for all hypotheses (among the top 100) that had a probability below the set threshold, their probabilities would be set to the value of the threshold, after which the probabilities of all the hypotheses would be re-normalized. The effect of this change was to allow the algorithm to adapt more quickly to changing input data. Some "tuning" of this modification is required to obtain best performance.

The second modification made was to change the way that hypothesis probability updating was performed for hypotheses containing targets with a single contributing track (single-track-per-target hypotheses). This modification was required for reasons that were essentially an outcome of limitations in the quality of the ionospheric and track data, as well as limitations in the modeling of the system. The major effect that caused problems was that for the single-track-per-target hypotheses, where the only difference is the propagation path assignment, after several updates one hypothesis would become much more probable than all of the others. If the data and models were a perfect representation of reality, this would not be a problem. However, because this is not the case, essentially selecting one of the contending hypotheses has often not been appropriate. The adjustment made was to use a modified likelihood update equation in place of equation 16. The replacement equation at this stage is equation 3 and is essentially an *ad hoc* modification. In practice this change makes the single-track-per-target hypotheses "fall-back" hypotheses, whose relative probability with respect to one another does not change with time (but may change with respect to other hypotheses), and whose probability will only become close to 1 if all other hypotheses are a poor match to the data. A more appropriate modification to

equation 16 may be found with further study, although the present modification appears adequate from a performance point of view.

As an indication of algorithm performance, Figures 7 and 8 show a typical multipath track scenario and the results of fusion using DMPTF over a total of 50 updates. Figure 7 shows track data received from the OTHR in radar coordinates (slant range and azimuth). Figure 8 shows the fused output of DMPTF in ground coordinates (range and azimuth) using the simplified two-layer ionospheric model, resulting in four possible propagation paths for each track. In the ionospheric model, the height of the E layer was set to 100 km with a standard deviation of 10 km and the F layer height was set to 300 km with a standard deviation of 20 km. The underlying truth of this example is represented by Figure 9, which shows track data received from a microwave radar for two targets travelling from the bottom right to the top left of the figure. Before continuing, let us also define the two letter notation used in the following text to denote the highest probability propagation path assignment made by the DMPTF algorithm. The first letter denotes the ionospheric path taken by the transmitted signal (ie, from the radar to the target) and the second letter denotes the return path (ie, from the target to the radar). For example an EF propagation path assignment represents transmission via the ionospheric E layer and a return signal propagating via the F layer.

The input data to the DMPTF algorithm (Figure 7) consists of seven tracks in total. Tracks 1, 2 and 3 are due to target 1 in Figure 9 and tracks 4, 5, 6 and 7 due to target 2. Note that the seven tracks do not all exist for the entire duration of the example. Tracks 1, 2, 4 and 5 exist for the entire duration while track 3 exists at the beginning but drops out towards the end. Track 7 starts a little way into the example, and track 6 starts a little later still; both tracks then run to the end of the duration of the example.

Figure 8 displays the fused target estimates of the highest probability path-dependent hypothesis at each update; hence the figure does not show the same hypothesis for all updates. The first feature that should be apparent about the result is that there are four distinct tracks shown in the figure, describing the two targets that actually exist. The example illustrates, as will be explained below, that with typical real data the algorithm may not always deliver a perfect result, but the results that DMPTF does give are generally consistent with the sometimes limited and ambiguous data that is available. The example was, in fact, chosen because it highlighted some of the ambiguities that occur in common scenarios, which can sometimes lead to more than one "correct" track to target assignment. Note also that the azimuth scale has been greatly expanded relative to the range scale to highlight differences between target positions when path assignments change. The scales have not been marked on the axes, however, to maintain an "unclassified" classification for this document.

Target 1 in Figure 8 is the result of fusing tracks 1, 2 and 3 from Figure 7, the propagation path assignments being FF, EF, and EE respectively. This target is consistent with the underlying truth for the example, ie, it matches up with target 1 of Figure 9. This consistency is maintained throughout the duration of the example, despite the switching of hypotheses associated with changes in propagation path assignments for tracks associated with target 2, as well as changes in the total number of tracks in the hypothesis tree.

Target 2 in Figure 8 starts in the bottom right-hand corner of the figure at the same time as target 1. Tracks 4 and 5, which are the only tracks that exist at this stage that are

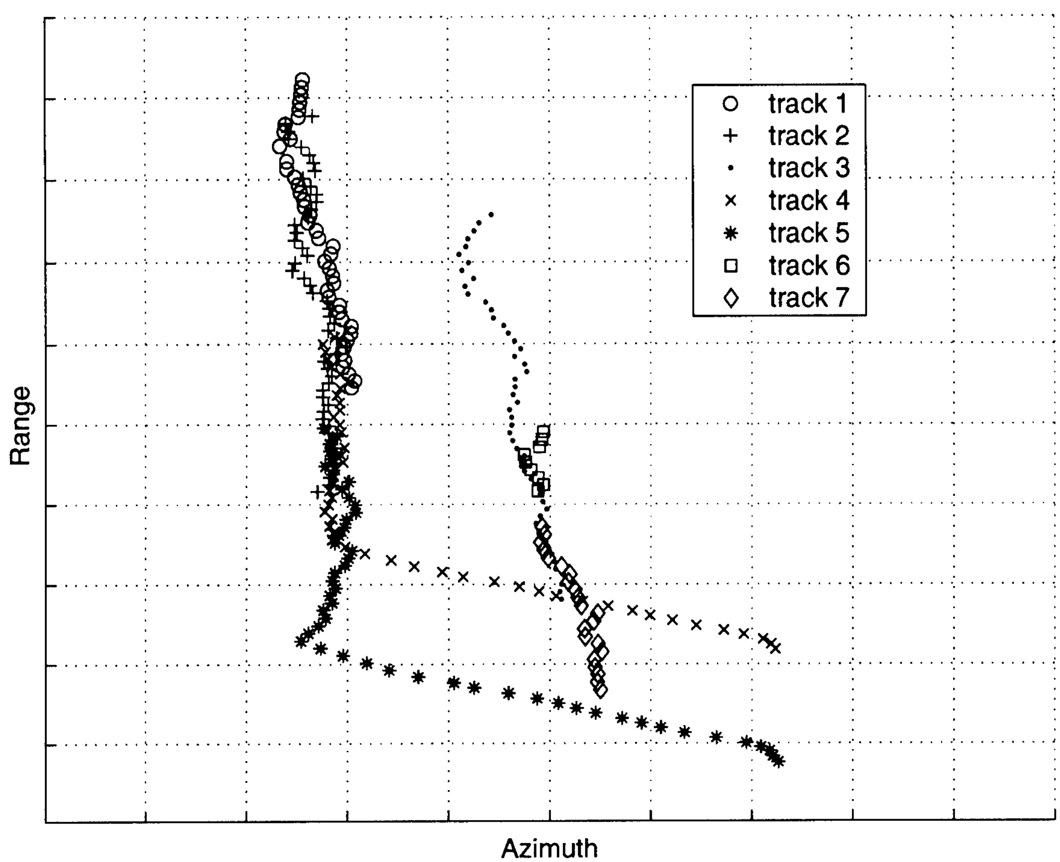

Figure 7: OTHR tracks in radar coordinates

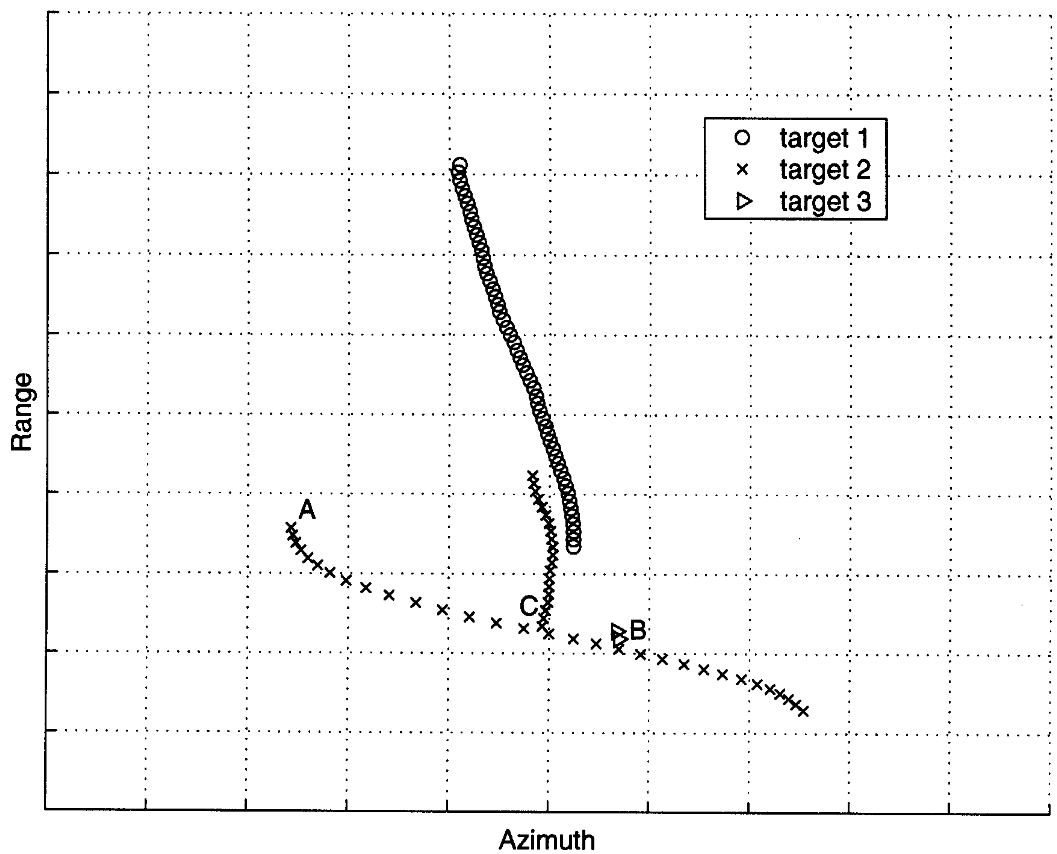

Figure 8: Fused OTHR tracks in ground coordinates

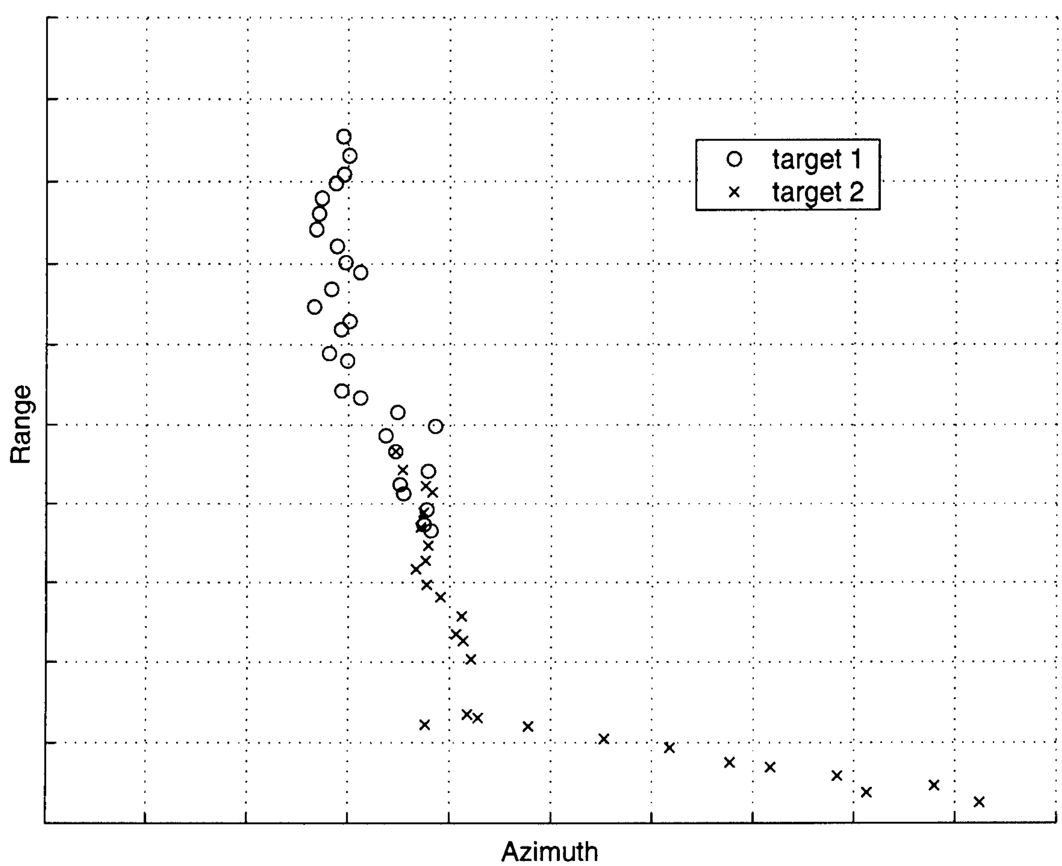

Figure 9: Microwave radar tracks

actually due to target 2, are fused together at the outset. The propagation assignments for tracks 4 and 5 are initially FE and EE, respectively. As becomes evident later in the example, these initial propagation path assignments are incorrect. The reason for this is that there are only two tracks available for association and fusion, resulting in ambiguity regarding which pair of propagation path assignments is appropriate for the target. This ambiguity is not due to the DMPTF algorithm itself, but results from the geometry of the propagation paths. For example, the relative positions of two tracks which are due to a single target, but propagating via the EE path and FE path respectively are very similar to those of two tracks propagating via EF and FF respectively. At the points labelled A and B a third track is initiated, track 7, which is originally determined to be a third target, (and assigned the EE path), but should be associated with tracks 4 and 5, ie, target 2. This incorrect target assignment is due to the fact that the path assignments for target 2 are initially incorrect. Due to the incorrect path assignments, the third track cannot initially be associated with target 2 in the highest probability hypothesis, so it as assigned to a third target. However, within two updates, the hypothesis that fuses tracks 4, 5 and 7, with path assignments of FF, EF and EE respectively, becomes the highest probability hypothesis, bringing the separate targets at A and B to C. The path assignments for both targets 1 and 2 are then correct. When track 6 is initiated at a later time, it is seamlessly fused into target 2 with path assignment FE, and when track 3 terminates, leaving target 1 with two tracks contributing to it and target 2 with four tracks contributing to it, the target estimates continue smoothly.

The example gives an indication of the difficulties associated with the environment in which the DMPTF algorithm must operate. The ambiguity between path assignments can sometimes result in an incorrect, but plausible, set of paths being assigned to a track. The effect of implementing the lower limit of probability for each hypothesis is also evident in this example, in that an originally incorrect assignment is rapidly resolved when more information, in the form of a third track, becomes available. Where the assignments are correct, the example shows that the DMPTF algorithm produces smooth, fused estimates of the targets in their correct spatial positions, despite losing and gaining track information as the example evolves over time.

In addition to implementation in the development test-bed, the DMPTF algorithm has been integrated into the JFAS radar developmental code for more thorough testing. To this end the radar's graphics displays were also modified to enable display of fused tracks that are produced by the DMPTF algorithm. Recently an extended trial of the algorithm has been performed on the JFAS radar, and a large amount of data collected for statistical analysis. While the statistical analysis has not been completed yet, early indications are that the algorithm performs well when good coordinate registration (CR) data is available. When the CR data is poor, as might be expected, incorrect associations can occur. Based on the testing to date, indications are that, at the present level of development of the DMPTF algorithm and the CR system, the DMPTF algorithm is capable of being used in a semi-automated fashion to give advice to an operator, but operator overseeing is still required. However, higher levels of automation can be expected with further development.

8 Track Association and Fusion using Measurements

There are important advantages in associating and fusing the measurements from which the tracker forms its tracks instead of fusing the tracker's estimates, ie, to use the tracker as a "selector" of measurements for the DMPTF algorithm. This approach provides key advantages of using measurements while still relieving DMPTF of the computational complexity associated with using all available measurements in the association and fusion process (including the responsibility of track initiation) as was done in [22]. The advantages come about from the reduced dependence of data that is used by the DMPTF algorithm when measurements are used. As was mentioned earlier in the report, track dependence occurs as a result of the following:

1. common process noise,
2. tracks being updated using common measurements, and
3. common propagation path segments.

Of the three causes of dependence above, the first does not occur at all when measurements are used in place of track estimates. The second source of dependence can reasonably readily be removed by checking for common measurements during the fusion process and discarding multiple occurences of measurements prior to fusion. The third source of dependence still remains, and must be dealt with by other means. Hence, by fusing the measurements from which the tracker forms its tracks, two of the three sources of data dependence can be removed with no significant increase in computational complexity.

Let us now derive the association and fusion equations for the case where measurements are used. Note that it is quite similar to the derivation for the case where estimates are used which was given in earlier sections; the differences come in the details of the equations.

Consider the creation of the hypothesis tree and calculation of the hypothesis probabilities and fused estimates. As indicated above, the measurements that are used are those that the tracking algorithm uses to produce the estimates $\psi_j(0), \psi_j(1), \psi_j(2), \ldots$ For the purpose of the subsequent discussion, it will be assumed the tracking algorithm is a nearest-neighbour (NN) tracker [3]. The approach described below can easily be generalized for use with a PDA tracker. Now let us denote the measurements as $\delta_j(0), \delta_j(1), \delta_j(2), \ldots$ Note that because of the assumption of a NN tracker there is only one measurement per update, ie, for each estimate in slant coordinates there is only one associated measurement. Note firstly the difference between the track estimate vector and the measurement vector. The track estimate vector in slant coordinates is

$$\psi_j(k) = \begin{bmatrix} R_j \\ \dot{R}_j \\ A_j \\ \dot{A}_j \end{bmatrix}_k$$

The measurement vector in slant coordinates is:

$$\delta_j(k) = \begin{bmatrix} R_j \\ \dot{R}_j \\ A_j \end{bmatrix}_k$$

Consider first the creation of the hypothesis tree at time $k = 0$. The derivations will follow a very similar line to that for the case where track estimates are used. Note also that, as was done in section 2 the time index (0) will be omitted for time $k = 0$. From the first measurement, δ_1, the propagation path transformations can be used to calculate the corresponding measurement in ground coordinates $z_1^{m_1}$ and its covariance $R_1^{m_1}$ for each propagation path $m_1 = 1, .., M_1$. Under the assumption that there are no false tracks, there is only one target association possible, namely target 1. The probability of each of the path dependent hypotheses is then:

$$P\left\{\lambda_1^{m_1} \left| \delta_1 \right.\right\} = \beta_1^{m_1} \tag{23}$$

where $\beta_j^{m_j} \triangleq P\left\{\theta_j^{m_j}\right\}$ and $\theta_j^{m_j}$ denotes the event that track τ_j is propagating via propagation path m_j. As for the case of using track estimates, the $\beta_j^{m_j}$ are *prior* probabilities which are estimated using physical measurements of the ionosphere.

For each subsequent measurement, δ_j, $j = 2, .., J$, the following recursive equation for calculating the probability of $\lambda_{n_1..n_K}^{m_1..m_K}$ from $\lambda_{n_1..n_{K-1}}^{m_1..m_{K-1}}$ can be derived in a similar fashion to that used when track estimates were considered:

$$P\left\{\lambda_{n_1..n_K}^{m_1..m_K} \left| \Delta^K \right.\right\} = \frac{\Lambda_K^{m_K} \beta_K^{m_K} P\left\{\lambda_{n_1..n_{K-1}}^{m_1..m_{K-1}} \left| \Delta^{K-1} \right.\right\}}{\sum\limits_{\bar{n}_K=1}^{B_{n_1..n_{K-1}}} \sum\limits_{\substack{\bar{m}_K=1, \\ \bar{m}_K \notin S_K}}^{M_K} \Lambda_K^{\bar{m}_K} \beta_K^{\bar{m}_K}} \tag{24}$$

where

$$\begin{aligned} \Lambda_K^{m_K} &\triangleq p\left(z_K^{m_K} \left| z_1^{m_1}, .., z_{K-1}^{m_{K-1}}, \lambda_{n_1..n_K}^{m_1..m_K} \right.\right) \\ \Lambda_K^{\bar{m}_K} &\triangleq p\left(z_K^{\bar{m}_K} \left| z_1^{m_1}, .., z_{K-1}^{m_{K-1}}, \lambda_{n_1..n_{K-1}\bar{n}_K}^{m_1..m_{K-1}\bar{m}_K} \right.\right) \end{aligned}$$

and $\Delta^K \triangleq \delta_1, .., \delta_K$

If $n_K \neq n_j$ for all $j = 1, .., K-1$, that is δ_K represents a new target, there is no prior information regarding the state of $z_K^{m_K}$. Hence the likelihood, $\Lambda_K^{m_K}$, is given by

$$\Lambda_K^{m_K} = \frac{1}{V_m} \tag{25}$$

where V_m is in this case the *"volume" in measurement space* in which the target may be.

If $n_K = n_j$ for one or more of $j = 1, .., K-1$, then δ_K represents the same target as at least one previously hypothesized target. In this case the new information contained in δ_j must be fused with the previous estimates. Assume that the hypothesis $\lambda_{n_1..n_{K-1}}^{m_1..m_{K-1}}$ is associated with T targets, $1 \leq T \leq K-1$, that the target corresponding to the track τ_K is t_i, $1 \leq i \leq T$, and that prior to considering $z_K^{m_K}$ there have been h track estimates

associated with target t_i for the current time instant. Let the previous estimate of the state of target t_i be $\bar{x}_i^h$ and its covariance $\bar{P}_i^h$. Since it is assumed that $z_K^{m_K}$ (with covariance $R_K^{m_K}$) is from target t_i the best prediction (the expected value) of $z_K^{m_K}$ is

$$E\left\{z_K^{m_K} \left| z_1^{m_1}, .., z_{K-1}^{m_{K-1}}, \lambda_{n_1..n_K}^{m_1..m_K} \right.\right\} = \bar{z}_i^h$$

where

$$\bar{z}_i^h = H\bar{x}_i^h$$

The covariance of $\left(z_K^{m_K} - \bar{z}_i^h\right)$ is:

$$\begin{aligned} S_K^{m_K} &\triangleq E\left\{\left[z_K^{m_K} - \bar{z}_i^h\right]\left[z_K^{m_K} - \bar{z}_i^h\right]'\right\} \\ &= E\left\{\left[\left(z_K^{m_K} - Hx_i\right) - H\left(\bar{x}_i^h - x_i\right)\right]\left[\left(z_K^{m_K} - Hx_i\right) - H\left(\bar{x}_i^h - x_i\right)\right]'\right\} \end{aligned}$$

Let $\tilde{x}_i^h = x_i - \bar{x}_i^h$ and $\tilde{z}_K^{m_K} = Hx_i - z_K^{m_K}$, where x_i is the true state of target t_i, then

$$\begin{aligned} S_K^{m_K} &= E\left\{\left[H\tilde{x}_i^h - \tilde{z}_K^{m_K}\right]\left[H\tilde{x}_i^h - \tilde{z}_K^{m_K}\right]'\right\} \\ &= E\left\{H\tilde{x}_i^h\left(\tilde{x}_i^h\right)'H' - H\tilde{x}_i^h\left(\tilde{z}_K^{m_K}\right)' - \tilde{z}_K^{m_K}\left(\tilde{x}_i^h\right)'H' + \tilde{z}_K^{m_K}\left(\tilde{z}_K^{m_K}\right)'\right\} \\ &= E\left\{H\tilde{x}_i^h\left(\tilde{x}_i^h\right)'H'\right\} - E\left\{H\tilde{x}_i^h\left(\tilde{z}_K^{m_K}\right)'\right\} - E\left\{\tilde{z}_K^{m_K}\left(\tilde{x}_i^h\right)'H'\right\} + E\left\{\tilde{z}_K^{m_K}\left(\tilde{z}_K^{m_K}\right)'\right\} \end{aligned}$$

Now

$$\begin{aligned} \bar{P}_i^h &\triangleq E\left\{\tilde{x}_i^h\left(\tilde{x}_i^h\right)'\right\} \\ R_K^{m_K} &\triangleq E\left\{\tilde{z}_K^{m_K}\left(\tilde{z}_K^{m_K}\right)'\right\} \end{aligned}$$

and let

$$\begin{aligned} U_{iK}^{hm_K} &\triangleq E\left\{\tilde{x}_i^h\left(\tilde{z}_K^{m_K}\right)'\right\} \\ U_{Ki}^{m_K h} &\triangleq E\left\{\tilde{z}_K^{m_K}\left(\tilde{x}_i^h\right)'\right\}. \end{aligned}$$

Then

$$S_K^{m_K} = H\bar{P}_i^h H' - HU_{iK}^{hm_K} - U_{Ki}^{m_K h}H' + R_K^{m_K} \tag{26}$$

The likelihood of $z_K^{m_K}$ (assuming a Gaussian distribution) is then given by

$$\Lambda_K^{m_K} = \left|2\pi S_K^{m_K}\right|^{-\frac{1}{2}} \exp\left[-\frac{1}{2}\left(\nu_K^{m_K}\right)'\left(S_K^{m_K}\right)^{-1}\nu_K^{m_K}\right] \tag{27}$$

where

$$\nu_K^{m_K} \triangleq \left(z_K^{m_K} - \bar{z}_i^h\right)$$

Consider now the estimate $\bar{x}_i^{h+1}$ of x_i and its covariance $\bar{P}_i^{h+1}$ obtained by the fusion of $\bar{x}_i^h, h = 1,2,..,H-1$ with $z_K^{m_K}$. To derive this we shall again use the fundamental equations of linear estimation as shown on pages 44 and 125 of [4] and reproduced in equations 6 in section 2. Hence, replacing the terms in the fundamental equations of linear estimation with their equivalent terms in this problem, and following the procedure used in section 2 to derive equations 7 and 8, results in the following fusion equations:

$$\bar{x}_i^{h+1} = \bar{x}_i^h + \left(\bar{P}_i^h H' - U_{iK}^{hm_K}\right)\left(S_K^{m_K}\right)^{-1}\left(z_K^{m_K} - \bar{z}_i^h\right) \tag{28}$$

$$\bar{P}_i^{h+1} = \bar{P}_i^h - \left(\bar{P}_i^h H' - U_{iK}^{hm_K}\right)\left(S_K^{m_K}\right)^{-1}\left(H\bar{P}_i^h - U_{Ki}^{m_K h}\right) \tag{29}$$

Now let us consider updates $k = 1,2,3,...$. As before, the hypothesis tree built up at $k = 0$ can be used as the basis for the tree at subsequent updates. The derivations again follow a similar line to those for the case of using track estimates. In order to shorten notation the symbol $\Delta^j(k)$ is defined such that $\Delta^j(k) \triangleq \delta_1(k),..,\delta_j(k)$.

In the creation of the hypothesis tree (at update $k = 0$) we derived the probability $P\left\{\lambda_{n_1..n_J}^{m_1..m_J}\left|\Delta^J(0)\right.\right\}$ of each path dependent hypothesis $\lambda_{n_1..n_J}^{m_1..m_J}$, $m_j \in \{1,2,..,M_j\}$, $n_j \in \left\{1,2,..,B_{n_1..n_{j-1}}\right\}$ given all the data available at update $k = 0$, ie the measurements $\delta_1(0),..,\delta_J(0)$. Now let us define the following shorthand notation

$$\begin{aligned} D^k &= \delta_1(0),..,\delta_J(0),\delta_1(1),..,\delta_J(1),....,\delta_1(k-1),..,\delta_J(k-1),\delta_1(k),..,\delta_J(k) \\ &= \Delta^J(0),\Delta^J(1),..,\Delta^J(k) \end{aligned}$$

ie, D^k represents all the track data, in radar coordinates, (measurements in this case) available up to and including update k . Hence

$$P\left\{\lambda_{n_1..n_J}^{m_1..m_J}\left|D^k\right.\right\} \triangleq P\left\{\lambda_{n_1..n_J}^{m_1..m_J}\left|\Delta^J(0),\Delta^J(1),..,\Delta^J(k)\right.\right\}$$

Consider the updating of the probability of the hypothesis $\lambda_{n_1..n_J}^{m_1..m_J}$ when the first measurement for update k , (ie, $\delta_1(k)$) is considered. Using Bayes rule we have

$$\begin{aligned} P\left\{\lambda_{n_1..n_J}^{m_1..m_J}\left|\delta_1(k),D^{k-1}\right.\right\} &= \frac{p\left(\delta_1(k)\left|\lambda_{n_1..n_J}^{m_1..m_J},D^{k-1}\right.\right)P\left\{\lambda_{n_1..n_J}^{m_1..m_J}\left|D^{k-1}\right.\right\}}{p\left(\delta_1(k)\left|D^{k-1}\right.\right)} \\ &= \frac{p\left(\delta_1(k)\left|\lambda_{n_1..n_J}^{m_1..m_J},D^{k-1}\right.\right)P\left\{\lambda_{n_1..n_J}^{m_1..m_J}\left|D^{k-1}\right.\right\}}{\sum\limits_{\bar{n}_J=1}^{B_{n_1..n_{J-1}}}..\sum\limits_{\bar{n}_1=1}^{1}\sum\limits_{\bar{m}_J=1}^{M_J}..\sum\limits_{\bar{m}_1=1}^{M_1}p\left(\delta_1(k)\left|\lambda_{\bar{n}_1..\bar{n}_J}^{\bar{m}_1..\bar{m}_J},D^{k-1}\right.\right)P\left\{\lambda_{\bar{n}_1..\bar{n}_J}^{\bar{m}_1..\bar{m}_J}\left|D^{k-1}\right.\right\}} \end{aligned}$$

Now, as noted earlier, in the implementation of MPTF the ground co-ordinate measurements $z_j^{m_j}(k)$ are used to perform the probability calculations. Again, wherever we encounter the conditioning event $\theta_j^{m_j}$ we replace $\delta_j(k)$ with $z_j^{m_j}(k)$. Now, when we have the event $\lambda_{n_1..n_j..n_J}^{m_1..m_j..m_J}$ we note that this includes the event $\theta_j^{m_j}$. Hence the above equation

can be replaced by

$$P\left\{\lambda_{n_1..n_J}^{m_1..m_J}\left|\delta_1(k),D^{k-1}\right.\right\} = \frac{p\left(z_1^{m_1}(k)\left|\lambda_{n_1..n_J}^{m_1..m_J},D^{k-1}\right.\right)P\left\{\lambda_{n_1..n_J}^{m_1..m_J}\left|D^{k-1}\right.\right\}}{\sum_{\bar{n}_J=1}^{B_{n_1..n_{J-1}}}..\sum_{\bar{n}_1=1}^{1}\sum_{\bar{m}_J=1}^{M_J}..\sum_{\bar{m}_1=1}^{M_1}p\left(z_1^{\bar{m}_1}(k)\left|\lambda_{\bar{n}_1..\bar{n}_J}^{\bar{m}_1..\bar{m}_J},D^{k-1}\right.\right)P\left\{\lambda_{\bar{n}_1..\bar{n}_J}^{\bar{m}_1..\bar{m}_J}\left|D^{k-1}\right.\right\}}$$

To calculate $p\left(z_1^{m_1}(k)\left|\lambda_{n_1..n_J}^{m_1..m_J},D^{k-1}\right.\right)$ we can again use information regarding the dynamics of the target that $z_1^{m_1}(k)$ is associated with. Now, as in section 6.1, let us assume that the motion of target t_i satisfies equation 13. Hence, given the fused estimate $\bar{x}_i^H(k-1)$ for the hypothesis being considered, where H is the number of track estimates that were combined at time $k-1$ to obtain $\bar{x}_i^H(k-1)$, we have (as in equations 14)

$$\bar{x}_i^0(k) = F(k-1)\,\bar{x}_i^H(k-1)$$

$$\bar{P}_i^0(k) = F(k-1)\,\bar{P}_i^H(k-1)\,F(k-1)' + Q_{k-1}$$

Using the above we can easily obtain the likelihood

$$\begin{aligned}\Lambda_1^{m_1}(k) &\triangleq p\left(z_1^{m_1}(k)\left|\lambda_{n_1..n_J}^{m_1..m_J},D^{k-1}\right.\right)\\ &= \mathcal{N}\left(z_1^{m_1}(k);\bar{z}_i^0(k),S_1^{m_1}(k)\right)\\ &= \left|2\pi S_1^{m_1}(k)\right|^{-\frac{1}{2}}\exp\left\{-\frac{1}{2}\left(z_1^{m_1}(k)-\bar{z}_i^0(k)\right)'S_1^{m_1}(k)^{-1}\left(z_1^{m_1}(k)-\bar{z}_i^0(k)\right)\right\}\end{aligned}$$

where

$$S_1^{m_1}(k) = H(k)\,\bar{P}_i^0(k)\,H(k)' - H(k)\,U_{i1}^{0m_1}(k) - U_{1i}^{m_10}(k)\,H(k)' + R_1^{m_1}(k)$$

with

$$\begin{aligned}\bar{P}_i^0(k) &\triangleq E\left\{\tilde{x}_i^0(k)\,\tilde{x}_i^0(k)'\right\}\\ R_1^{m_1}(k) &\triangleq E\left\{\tilde{z}_1^{m_1}(k)\,\tilde{z}_1^{m_1}(k)'\right\}\\ U_{i1}^{0m_1}(k) &\triangleq E\left\{\tilde{x}_i^0(k)\,\tilde{z}_1^{m_1}(k)'\right\}\\ U_{1i}^{m_10}(k) &\triangleq E\left\{\tilde{z}_1^{m_1}(k)\,\tilde{x}_i^0(k)'\right\}\end{aligned}$$

and

$$\begin{aligned}\tilde{x}_i^0(k) &= x_i(k)\,\bar{x}_i^0(k)\\ \tilde{z}_1^{m_1}(k) &= Hx_i(k) - z_1^{m_1}(k)\end{aligned}$$

Consider now, the updating of the probability of the hypothesis $\lambda_{n_1..n_j..n_J}^{m_1..m_j..m_J}$ when the j^{th} measurement for update k, (ie, $\delta_j(k)$, $1 < j \le J$) is considered. Using Bayes rule we have

$$\begin{aligned}&P\left\{\lambda_{n_1..n_J}^{m_1..m_J}\left|\Delta^j(k),D^{k-1}\right.\right\}\\ &= \frac{p\left(\delta_j(k)\left|\lambda_{n_1..n_J}^{m_1..m_J},\Delta^{j-1}(k),D^{k-1}\right.\right)P\left\{\lambda_{n_1..n_J}^{m_1..m_J}\left|\Delta^{j-1}(k),D^{k-1}\right.\right\}}{p\left(\delta_j(k)\left|\Delta^{j-1}(k),D^{k-1}\right.\right)}\\ &= \frac{p\left(\delta_j(k)\left|\lambda_{n_1..n_J}^{m_1..m_J},\Delta^{j-1}(k),D^{k-1}\right.\right)P\left\{\lambda_{n_1..n_J}^{m_1..m_J}\left|\Delta^{j-1}(k),D^{k-1}\right.\right\}}{\sum_{\bar{n}_J=1}^{B_{n_1..n_{J-1}}}..\sum_{\bar{n}_1=1}^{1}\sum_{\bar{m}_J=1}^{M_J}..\sum_{\bar{m}_1=1}^{M_1}\left[\begin{array}{c}p\left(\delta_j(k)\left|\lambda_{\bar{n}_1..\bar{n}_J}^{\bar{m}_1..\bar{m}_J},\Delta^{j-1}(k),D^{k-1}\right.\right)\times\\ P\left\{\lambda_{\bar{n}_1..\bar{n}_J}^{\bar{m}_1..\bar{m}_J}\left|\Delta^{j-1}(k),D^{k-1}\right.\right\}\end{array}\right]}\end{aligned}$$

Now, as earlier, wherever we encounter the conditioning event $\theta_j^{m_j}$ we can replace $\delta_j(k)$ with $z_j^{m_j}(k)$. Again, when we have the event $\lambda_{n_1..n_j..n_J}^{m_1..m_j..m_J}$ we note that this includes the event $\theta_j^{m_j}$, hence the above equation can be replaced by

$$P\left\{\lambda_{n_1..n_J}^{m_1..m_J}\left|\Delta^j(k), D^{k-1}\right.\right\} = \frac{p\left(z_j^{m_j}(k)\,|\lambda_{n_1..n_J}^{m_1..m_J}, \Delta^{j-1}(k), D^{k-1}\right) P\left\{\lambda_{n_1..n_J}^{m_1..m_J}\,|\Delta^{j-1}(k), D^{k-1}\right\}}{\sum_{\bar{n}_J=1}^{B_{n_1..n_{J-1}}} .. \sum_{\bar{n}_1=1}^{1} \sum_{\bar{m}_J=1}^{M_J} .. \sum_{\bar{m}_1=1}^{M_1} \left[\begin{array}{c} p\left(z_j^{\bar{m}_j}(k)\,|\lambda_{\bar{n}_1..\bar{n}_J}^{\bar{m}_1..\bar{m}_J}, \Delta^{j-1}(k), D^{k-1}\right) \times \\ P\left\{\lambda_{\bar{n}_1..\bar{n}_J}^{\bar{m}_1..\bar{m}_J}\,|\Delta^{j-1}(k), D^{k-1}\right\} \end{array}\right]}$$

Then we can easily obtain the likelihood

$$\begin{aligned}
\Lambda_j^{m_j}(k) &\triangleq p\left(z_j^{m_j}(k)\,|\lambda_{n_1..n_J}^{m_1..m_J}, \Delta^{j-1}(k), D^{k-1}\right) \\
&= \left|2\pi S_j^{m_j}(k)\right|^{-\frac{1}{2}} \exp\left\{-\frac{1}{2}\left(z_j^{m_j}(k) - \bar{z}_i^h(k)\right)' S_j^{m_j}(k)^{-1}\left(z_j^{m_j}(k) - \bar{z}_i^h(k)\right)\right\}
\end{aligned}$$

where

$$S_j^{m_j}(k) = H(k)\,\bar{P}_i^h(k)\,H(k)' - H(k)\,U_{ij}^{hm_j}(k) - U_{ji}^{m_jh}(k)\,H(k)' + R_j^{m_j}(k)$$

with

$$\begin{aligned}
\bar{P}_i^h(k) &\triangleq E\left\{\tilde{x}_i^h(k)\,\tilde{x}_i^h(k)'\right\} \\
R_j^{m_j}(k) &\triangleq E\left\{\tilde{z}_j^{m_j}(k)\,\tilde{z}_j^{m_j}(k)'\right\} \\
U_{ij}^{hm_j}(k) &\triangleq E\left\{\tilde{x}_i^h(k)\,\tilde{z}_j^{m_j}(k)'\right\} \\
U_{ji}^{m_jh}(k) &\triangleq E\left\{\tilde{z}_j^{m_j}(k)\,\tilde{x}_i^h(k)'\right\} \\
\tilde{x}_i^h(k) &= x_i(k) - \bar{x}_i^h(k) \\
\tilde{z}_j^{m_j}(k) &= Hx_i(k) - z_j^{m_j}(k)
\end{aligned}$$

and $\bar{x}_i^h(k)$ is the most recent fused estimate/prediction for target t_i for the hypothesis being considered.

Hence combining the cases of $j = 1$, $1 < j \leq J$ and again assuming that no two resolved ground tracks which are due to the same target can be associated with the same propagation path gives the following probability update for the hypothesis $\lambda_{n_1..n_j..n_J}^{m_1..m_j..m_J}$ using the j^{th} measurement of update k, $\delta_j(k)$:

$$P\left\{\lambda_{n_1..n_J}^{m_1..m_J}\left|\Delta^j(k), D^{k-1}\right.\right\} = \frac{\Lambda_j^{m_j}(k)\,P\left\{\lambda_{n_1..n_J}^{m_1..m_J}\,|\Delta^{j-1}(k), D^{k-1}\right\}}{\sum_{\bar{n}_1=1}^{1} .. \sum_{\bar{n}_J=1}^{B_{n_1..n_{J-1}}} \sum_{\bar{m}_1=1}^{M_1} .. \sum_{\substack{\bar{m}_J=1 \\ \bar{m}_J \notin S_J}}^{M_J} \Lambda_j^{\bar{m}_j}(k)\,P\left\{\lambda_{\bar{n}_1..\bar{n}_J}^{\bar{m}_1..\bar{m}_J}\,|\Delta^{j-1}(k), D^{k-1}\right\}} \tag{30}$$

where, for $j = 1$:

$$\begin{aligned}
\Lambda_j^{m_j}(k) &= \Lambda_1^{m_1}(k) \triangleq p\left(z_1^{m_1}(k)\,|\lambda_{n_1..n_J}^{m_1..m_J}, D^{k-1}\right) \\
\Lambda_j^{\bar{m}_j}(k) &= \Lambda_1^{\bar{m}_1}(k) \triangleq p\left(z_1^{\bar{m}_1}(k)\,|\lambda_{\bar{n}_1..\bar{n}_J}^{\bar{m}_1..\bar{m}_J}, D^{k-1}\right)
\end{aligned}$$

and for $1 < j \leq J$:

$$\begin{aligned}
\Lambda_j^{m_j}(k) &\triangleq p\left(z_j^{m_j}(k)\,|\lambda_{n_1..n_J}^{m_1..m_J}, \Delta^{j-1}(k), D^{k-1}\right) \\
\Lambda_j^{\bar{m}_j}(k) &\triangleq p\left(z_j^{\bar{m}_j}(k)\,|\lambda_{\bar{n}_1..\bar{n}_J}^{\bar{m}_1..\bar{m}_J}, \Delta^{j-1}(k), D^{k-1}\right)
\end{aligned}$$

The likelihood for both cases is then

$$\Lambda_j^{m_j}(k) = \left|2\pi S_j^{m_j}(k)\right|^{-\frac{1}{2}} \exp\left[-\frac{1}{2}\nu_j^{m_j}(k)' S_j^{m_j}(k)^{-1} \nu_j^{m_j}(k)\right] \tag{31}$$

where

$$\nu_j^{m_j}(k) \triangleq \left(z_j^{m_j}(k) - \bar{z}_i^h(k)\right)$$

and

$$\begin{aligned}
\bar{P}_i^h(k) &\triangleq E\left\{\tilde{x}_i^h(k)\,\tilde{x}_i^h(k)'\right\} \\
R_j^{m_j}(k) &\triangleq E\left\{\tilde{z}_j^{m_j}(k)\,\tilde{z}_j^{m_j}(k)'\right\} \\
U_{ij}^{hm_j}(k) &\triangleq E\left\{\tilde{x}_i^h(k)\,\tilde{z}_j^{m_j}(k)'\right\} \\
U_{ji}^{m_jh}(k) &\triangleq E\left\{\tilde{z}_j^{m_j}(k)\,\tilde{x}_i^h(k)'\right\} \\
\tilde{x}_i^h(k) &= x_i(k) - \bar{x}_i^h(k) \\
\tilde{z}_j^{m_j}(k) &= Hx_i(k) - z_j^{m_j}(k)
\end{aligned}$$

and $\bar{x}_i^h(k)$ is the most recent fused estimate/prediction for target t_i that $z_j^{m_j}(k)$ is assumed to be associated with for the hypothesis being considered.

Consider now the estimate $\bar{x}_i^{h+1}(k)$ of $x_i(k)$ and its covariance $\bar{P}_i^{h+1}(k)$ obtained by the fusion of $\bar{x}_i^h(k), h = 0, 1, .., H-1$ with $z_j^{m_j}(k)$. This is again derived using the fundamental equations of linear estimation and essentially the same reasoning as shown earlier for $k = 0$. The resulting fusion equations are

$$\bar{x}_i^{h+1}(k) = \bar{x}_i^h(k) + \left(\bar{P}_i^h(k)\,H(k)' - U_{ij}^{hm_j}(k)\right) S_j^{m_j}(k)^{-1}\left(z_j^{m_j}(k) - \bar{z}_i^h(k)\right) \tag{32}$$

$$\bar{P}_i^{h+1}(k) = \bar{P}_i^h(k) - \left(\bar{P}_i^h(k)\,H(k)' - U_{ij}^{hm_j}(k)\right) S_j^{m_j}(k)^{-1}\left(H(k)\,\bar{P}_i^h(k) - U_{ji}^{m_jh}(k)\right) \tag{33}$$

9 Fusion of Microwave Radar and Multiple OTHR tracks

During the development of the DMPTF algorithm, it was always kept in mind that the algorithm, if possible, should be extendable to multi-sensor applications thus giving a single coherent algorithm for fusing multipath OTHR tracks and tracks from other sources such as microwave radar and GPS. This aim was in fact achieved, so that microwave radar tracks (or GPS tracks) can be easily incorporated into the fusion algorithm by treating estimates from the microwave radar in a similar manner to the OTHR tracks.

Some properties of the microwave radar tracks that should be noted, are

- The tracks generally only have a single propagation path associated with them (except in occasional anomalous circumstances), hence any two tracks from the same microwave radar will not, in general, be from the same target.
- The microwave radars are generally not co-located with the OTHR hence transformations need to be provided to a common ground coordinate system.
- Covariance matrices for the microwave track estimates are often not available in legacy systems, hence approximations of these covariance matrices may need to be made.

Let us now consider how microwave radar tracks would be incorporated into the hypothesis tree. Essentially the microwave radar tracks can be treated almost identically to the OTHR tracks. The microwave radar tracks arrive asynchronously with respect to the OTHR tracks; however, this is simply accommodated through the use of the predictions that are produced by the MPTF target model. For example, if, say, two OTHR tracks result in the creation of the hypothesis tree at update $k = 0$ and a microwave radar track is introduced at time $k = 1$ then the tree at $k = 0$ is simply extrapolated to $k = 1$ and then extended to 3 tracks using the microwave radar data. Figure 10 shows the path independent hypothesis tree for this example. The path dependent tree will of course incorporate the number of paths occuring with each of the OTHR tracks and only one "path" for the microwave radar track. The "path" for the microwave radar track is the transformation from the microwave radar's coordinates to the common ground coordinates that are used.

The code for incorporating microwave radar tracks into the DMPTF algorithm has been implemented in the test-bed, and is expected to be tested in the near future on real track data. The main extension that was required was to derive the transformations for the microwave radar tracks to the common ground coordinates. The JFAS ground coordinates were chosen to be the common coordinate system, ie, range and azimuth relative to the receive array, with the receive array boresight corresponding to zero azimuth.

The same principles that have been applied to fusing the OTHR multipath tracks with microwave radar tracks apply to fusion of OTHR and GPS tracks, as well as fusion of multipath tracks from multiple OTHRs in an overlapping network. With GPS tracks the method of fusion with OTHR tracks is essentially identical to that for fusion of microwave radar and OTHR tracks. In the case of multiple OTHRs, each OTHR will have a different

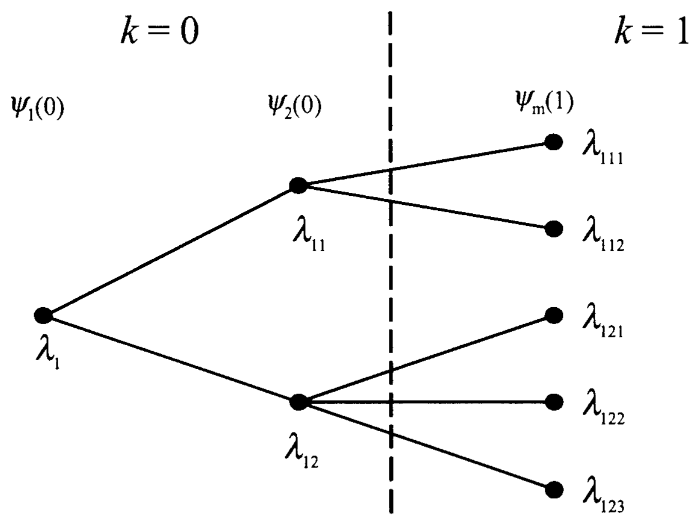

Figure 10: Path independent hypothesis tree for two OTHR tracks at time $k = 0$ and one microwave radar track at time $k = 1$

set of multiple paths, but this is easily accommodated in the DMPTF algorithm. With regard to coordinate systems, a common ground coordinate system will be required as opposed to the polar coordinates referenced to JFAS that is used at present. In principle this poses no difficulties other than the requirement for more computation to be performed to convert to the common coordinates.

A point worth making here is that, in general, microwave radar tracks and GPS tracks are much more accurate than OTHR tracks. This fact can be used to improve coordinate registration (CR) of the OTHR where microwave radar or GPS tracks are available. For example, if the registration of the OTHR is initially good enough to perform correct association of a group of multipath OTHR tracks with, say, their corresponding microwave radar track, then the offset between the OTHR tracks and the microwave radar track can be used to perform corrections to the OTHRs slant to ground coordinate transformations. Because the height of ionospheric layers can be expected to not change abruptly, the corrections will be able to be utilized for regions around the tracks as well. The extent of these regions, and the degradation in the accuracy of the CR corrections as a function of distance from the registered tracks would need to be determined by experiment.

10 Summary and Conclusions

A dynamic multipath fusion (DMPTF) algorithm which is capable of fusing over-the-horizon radar (OTHR) multipath tracks and non-OTHR tracks (e.g. microwave radar or GPS), as well as dealing with multipath tracks from OTHR networks is presented in this report. The algorithm achieves this through a very general model based approach which can deal with both the multipath effects within a single OTHR as well as asynchronicity between multiple sensors. Important advances of the algorithm presented in this report over earlier related work, ie, [17], [18] and [19] are:

- The new algorithm can associate and fuse asynchronous track data, thus making combined multisensor-multipath track fusion possible; particularly of interest is the algorithm's direct applicability to fusion with microwave radar, GPS and multiple OTHR tracks.
- The new algorithm takes into account temporal relationships between multipath and multisensor tracks as well as the instantaneous spatial relationships. This is of theoretical importance in addition to benefitting performance.
- An adjunct algorithm which can effectively deal with increases and decreases in the number of tracks with time in DMPTF's hypothesis tree has been developed. This is important for achieving temporally consistent association hypotheses and fused tracks. Without this, each time a multipath track drops out, a new hypothesis tree would need to be constructed, losing all the information in the previous tree.
- An effective and efficient pruning algorithm, as well as a clustering algorithm have been developed which together enable association and fusion of very large numbers of tracks. This has been tested on the JFAS OTHR in an operational setting showing that the DMPTF algorithm together with the associated pruning and clustering can successfully deal with all the tracks in the entire coverage area of the radar in real time.
- A set of equations have been derived for the association and fusion of the measurements from which the multipath and multisensor tracks are created. This enables an alternative fusion approach where some sources of track dependence are avoided.

The DMPTF algorithm has been implemented in C++ in a test-bed, for trialling with simulated and real data, and also implemented as a prototype on the JFAS OTHR for testing in an operational setting and for collection of data for statistical analysis. To date, testing has centred on multipath data from the JFAS radar. Code has also been implemented for fusion with microwave radar tracks which awaits testing. Considerable effort was invested in the development of the C++ code for DMPTF to achieve modularity, expandability, and maintainability, and a semi-automated software documentation program was implemented. As an outcome, it should be relatively easy to further develop the code and transfer it to another target system such as JORN if required.

With regard to performance, testing of DMPTF with OTHR multipath tracks (ie, real data) has been performed on a development test-bed from early stages in its development.

Recently an extended trial of the algorithm has been performed on the JFAS radar, and a large amount of data collected for statistical analysis. While the statistical analysis has not yet been completed, early indications are that the algorithm performs well when good coordinate registration (CR) data is available. However, when the CR data is poor, incorrect associations often occur. Hence, at the present level of development of DMPTF and the CR system, the DMPTF algorithm is capable of being used in a semi-automated fashion to give advice to an operator, but operator overseeing is still required. Higher levels of automation can be expected with further development of both DMPTF and CR.

Further operational assessment and development is strongly recommended. In particular:

- The code for automatically associating and fusing multipath OTHR tracks should be further operationally assessed and enhanced.
- The code for fusion of OTHR and microwave radar tracks which has been implemented should be assessed.
- The fusion of tracks in OTHR networks in regions of overlapping surveillance coverage should be implemented and assessed.
- The potential for improving OTHR coordinate registration accuracy via feedback from other surveillance sensors should be developed.
- Further research on track dependence and development of a realistic ionospheric model (the two are related) should bear fruit with regard to improving performance.

Authors' Contributions

The contributions of the authors were as follows:

Peter Sarunic performed the first principles derivation of the static MPTF, and proposed and led the development of the new pruning algorithm. He conceived and performed the mathematical derivations of the dynamic multipath track fusion algorithm, proposed the extension of DMPTF to perform fusion of selected measurements to reduce data dependence, and derived the equations for doing this. He also wrote the core of the C++ code for the fusion algorithms and led the subsequent coding and testing.

Kruger White, who had been working on multipath fusion for some time prior to the work presented in this report, provided much useful information and advice based on his earlier work on the problem, including advice on limitations he had encountered with the earlier fusion algorithm. He also developed coordinate transformation software, participated in literature surveys, particularly on track dependence, and proposed the investigation of the Covariance Intersection technique.

Mark Rutten joined the team shortly after the commencement of the work which is the subject of this report. After Peter's original coding of the static MPTF code in C++, Mark coded extensions to include pruning, target dynamics and clustering. He contributed to the development of the K-best search algorithm and developed the theoretical aspects of clustering (not covered in this report). He also contributed to testing of algorithms.

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The authors would like to thank Peter den Hartog and Kim Kieu for their efforts in the implementation and testing of the DMPTF software. Peter den Hartog performed integration of the software with the JFAS radar, organised trials, and did some of the graphical display work for the test-bed software. He also developed a web-site for presentation of algorithm developments. Kim did the major part of the display graphics for the test-bed and the integration of DMPTF with the graphics software. Both Kim Kieu and Peter den Hartog, contributed to the testing of the fusion algorithms and associated software. Their efforts are much appreciated. While John Percival did not participate significantly in the work presented in this report, he played a key role in the earlier work on Multipath Track Fusion which is referenced in this report, for which his efforts are recognized.

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Appendix A: Summary of Errors in Previously Derived MPTF Equations

This appendix summarizes errors found in some of the static equations presented in [17] and [19]. The equations were determined by analogy arguments from multisensor fusion work published in the literature; however, the translation to the multi-path fusion problem domain incurred some mistakes. The equations in question are the counterparts of equations 2, 3, and 5 in section 2 of this report. In [17] and [19] two probability update equations are presented. The equations are presented below using the notation of [19]. The reader is referred to the papers in question for a detailed description of the notation, as well as a description of how the equations were obtained.

$$\Pr\{\lambda|\bar{Z}^k \cup Z_i^k, D^k\} = \tag{A1}$$
$$\begin{cases} C^{-1}\Pr\{\bar{\lambda}|\bar{Z}^k, D^k\}\ \beta_i^{(m_i)}(k), & \tau_i^{(m_i)} \in \lambda,\quad \tau_i^{(m_i)} \notin \bar{\lambda}, \\ C^{-1}\Pr\{\bar{\lambda}|\bar{Z}^k, D^k\}\ \beta_i^{(m_i)}(k)\ L(\tau^{(\bar{m},m_i)}|\bar{Z}^k, Z_i^k, D^k), & \tau^{(\bar{m},m_i)} = \bar{\tau}^{(\bar{m})} \cup \tau_i^{(m_i)} \in \lambda \end{cases}$$

The first equation is the probability update for a track estimate which is assumed to be due to a new target for the hypothesis in question. The second is the probability update for a track estimate that represents the same target as at least one previously hypothesized target.

The second of equations A1 has a likelihood term which in reference [17] is obtained using the following relationship

$$L(\tau_1^{(m_1)} \cup \tau_2^{(m_2)}|\tilde{Z}_1^k \cup \tilde{Z}_2^k) = \tag{A2}$$
$$\left|2\pi\left(P_{y_1}^{(m_1)} + P_{y_2}^{(m_2)}\right)\right|^{-\frac{1}{2}}\exp\left[-\frac{1}{2}\left(\hat{y}_1^{(m_1)} - \hat{y}_2^{(m_2)}\right)'\left(P_{y_1}^{(m_1)} + P_{y_2}^{(m_2)}\right)^{-1}\left(\hat{y}_1^{(m_1)} - \hat{y}_2^{(m_2)}\right)\right]$$

whereas in [19] the likelihood term is determined using the equation below

$$L(\tau^{(\bar{m},m_i)}|\bar{Z}^k, Z_i^k, D^k) = \rho^{-1}\,|2\pi\Pi|^{-\frac{1}{2}}\exp\left[-\frac{1}{2}\left(\hat{\bar{y}}^{(\bar{m})} - \hat{y}_i^{(m_i)}\right)'\Pi^{-1}\left(\hat{\bar{y}}^{(\bar{m})} - \hat{y}_i^{(m_i)}\right)\right] \tag{A3}$$

with

$$\Pi = P_{\bar{y}}^{(\bar{m})} + P_{y_i}^{(m_i)}$$

and where ρ is the *a priori* target density.

Firstly, in both of the probability update equations A1 there is a normalization constant C^{-1}. The way to determine its value is not given in either of the papers; however, in an implementation of the static MPTF algorithm based on the above papers, C was taken to be the *sum of the unnormalized* probabilities at level k of the hypothesis tree, ie,

$$C = \sum_{\substack{\text{All } \lambda \text{ at} \\ \text{level } k}} \Pr_{unnormalized}\{\lambda|\bar{Z}^k \cup Z_i^k, D^k\} \tag{A4}$$

This is, in fact, incorrect as can be seen from equation 2 in section 2 of this report.

Secondly, comparing the first equation with equations 2, 3 in section 2, shows that it is missing a $1/V_s$ term. This is of course important, as it affects the relative magnitude of the probabilities for new-target hypotheses when compared to hypotheses assuming an already existing target.

Thirdly, because equation A4 was mistakenly used for normalization, it made the second probability equation of equations A1 dimensionally inconsistent if the likelihood equation of reference [17] (ie, equation A2) is used for the likelihood term. To compensate for this, an adjustment was made to the likelihood equation (ie, adding the ρ^{-1} term, where ρ is an assumed target density), giving equation A3 which was presented in [19]. Unfortunately, this adjustment still did not correct the equations, as can be seen by comparison with equations 2 and 5 in section 2 of this report.

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P.W. Sarunic, K.A.B. White and M.G. Rutten

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19. ABSTRACT

Over-the-horizon radar (OTHR) and microwave radar networks can together generate track data over a wide surveillance region. However the data is often subject to ambiguity and uncertainty due to the complexities of the HF signal propagation environment, which give rise to multipath OTHR tracks, as well as ambiguities in target associations between multiple microwave radars. This report describes an association and fusion algorithm which deals with both sources of uncertainty. The algorithm is capable of fusing OTHR multipath tracks and non-OTHR tracks (e.g. microwave radar or GPS), as well as dealing with multipath tracks from OTHR networks. The algorithm achieves this through a very general, model based, approach which deals with multipath effects as well as asynchronicity between sources of data. Importantly, the approach incorporates track history in its computation of association probabilities and fused estimate calculations, thus exploiting temporal as well as instantaneous spatial relationships between tracks.

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